# Narrow linewidth measurement with a Fabry-Pérot interferometer using a length modulation technique



Master thesis

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## 1 Introduction

Narrow linewidth coherent light sources are necessary for fibre-optic sensors, spectroscopy, LIDAR, in coherent optical fibre communications and many more applications [1]. But how is the linewidth of a laser experimentally quantified? Indeed, this may be difficult.

In this thesis, the tested laser<sup>1</sup> has an estimated linewidth of a few 100 kHz and an output spectrum from 900 nm to 1300 nm [2]. This corresponds to a ratio of laser frequency to linewidth in the order of  $10^9$ . To determine a suitable measurement technique, an overview of existing techniques is given first.

The difficulty of measuring linewidths of lasers fundamentally depends on the linewidth itself and therefore on the necessary resolution. Large linewidths (i.e. > 10 GHz) are easily measurable with traditional techniques of optical spectrum analysis such as the use of diffraction gratings. Moreover, it is possible to convert frequency fluctuations into intensity fluctuations by using an optical frequency discriminator (i.e. a high-finesse reference cavity or an unbalanced interferometer), but in this case, the resolution is limited, too and confined to a few MHz [1].

To measure linewidths of cw-lasers in the sub-MHz region, heterodyne techniques are often used. Here, a beat note between the tested laser and a second laser with narrow linewidth is recorded with a photo diode to transform the frequencies from the optical into the radio regime, which can be measured with standard electronics. A disadvantage of this technique is the need of a second laser that deviates much less than 1 nm from the tested laser's wavelength. To avoid this, delayed self-heterodyne interferometry can be used. Therefore, the light of the tested laser is delayed by its coherence length with an optical fibre of several kilometres length and frequency shifted by an acousto-optic modulator superimposed again [1]. The downside of this technique is the fact that a narrow linewidth involves a high coherence length. A delay of about 25 km is necessary to measure linewidths in the domain of a few tens of kHz. This is not a problem in case of laser wavelengths where fibres only exhibit low attenuation, i.e. for the Telecom wavelengths  $1.3 \,\mu$ m and  $1.55 \,\mu$ m, but there is no optical fibre that can be used when it comes to shorter wavelengths [1]. Thus, in this case<sup>2</sup>, delayed self-heterodyne interferometry is only usable for the wavelengths close to 1300 nm.

Therefore, in this thesis a technique with the accuracy of delayed self-heterodyne interferometry but with wavelength compatibility down to 900 nm is needed.

The beat of the laser with a frequency comb, which is basically an extremely high resolution ruler in the frequency domain, would be a suitable technique of linewidth measurement. As the equipment is very expensive, another technique is to be developed [3][4][5].

Hence, in this thesis a high resolution Fabry-Pérot interferometer is built to measure the linewidth with a *cavity length modulation technique*. To realise that, one of the two cavity mirrors is mounted on a piezo electric transducer and swept in a way that the cavity transmission fringe periodically overlaps with the line shape of the tested laser in the frequency domain. The transmitted intensity is then analysed to determine the linewidth [6]. Besides, two other experiments are performed in order to characterize the cavity<sup>3</sup>. This is necessary for the analysis of the data from the linewidth measurement with the cavity length modulation technique.

In addition, an already existing delayed self-heterodyne setup is used to measure the linewidth at wavelengths at 1300 nm and below to compare the results with those acquired by the cavity length modulation technique. Therefore, the delayed self-heterodyne setup serves as a reference to check the built Fabry-Pérot interferometer.

This thesis is structured as follows:

In chapter (2) the theoretical fundamentals necessary for this thesis are discussed, starting in chapter (2.1) with the origin of a finite linewidth and the characteristics of a related lineshape.

<sup>&</sup>lt;sup>1</sup>As tested laser, the  $H\ddot{u}bner C-WAVE$  is used.

 $<sup>^{2}</sup>$ How far towards shorter wavelengths this technique can be used depends on the optical power of the tested laser, the attenuation in the fibre and the sensitivity of the used photo detector.

<sup>&</sup>lt;sup>3</sup>During this thesis the terms Fabry-Pérot interferometer and cavity are used synonymously.

Moreover, in chapter (2.2) the principles of the tested laser, an optical parametric oscillator, are presented. This is necessary for a theoretical investigation of the tested laser's expected linewidth, later.

In addition, in chapter (2.3) Gaussian beams that describe the mode structure of a cavity are discussed, since knowledge about Gaussian beams is of fundamental importance in order to design a stable Fabry-Pérot interferometer. Besides, if a laser beam is coupled into a cavity, the mode structures of incident beam and interferometer have to match (*mode-matching*). In an experiment, this is only realisable for known mode structures.

In chapter (2.4) the transmission profile, the (longitudinal and transverse) mode structure, the condition for a stable operation and expectable resolution of Fabry-Pérot interferometers is presented. Moreover, the design conditions to build up a Fabry-Pérot interferometer are given. Besides, the cavity length modulation technique using a Fabry-Pérot interferometer is introduced and the realisation of mode-matching is discussed.

Afterwards, since an acousto-optic modulator is of fundamental importance and used in several experiments, its theory is presented in chapter (2.5). Besides, an acousto-optic modulator double-pass configuration that solves the issue with a frequency dependent pointing of the acousto-optic modulator's first diffraction order is introduced.

In chapter (2.6), the principles of delayed self-heterodyne interferometry used as linewidth measurement technique (in addition to the cavity length modulation technique) are given.

Then, in chapters (3.1.1) and (3.1.2) the experimental setup and the performance of the cavity characterisation measurements and the linewidth measurement (using the cavity length modulation technique) are described respectively. Thereby, the results of the cavity characterisation experiments are necessary for the analysis of the cavity length modulation technique's data.

Subsequently, in chapters (3.2.1) and (3.2.2) the experimentally obtained data is analysed and interpreted.

Finally, the experimental results are summarised in chapters (3.3.1), (3.3.2) and (4) and an outlook for future work is given.

## 2 Theory

## 2.1 Linewidth of single-frequency lasers

The linewidth, even of a continuous-wave single-mode laser, can never be made perfectly monochromatic. The fundamental reason for this is spontaneous emission.

In figure (1) the fundamental lasing transitions absorption, stimulated emission and spontaneous emission in a two level system are shown, whereby absorption is not of interest in this case.

Since stimulated emission adds coherently, with a defined phase relationship to the lasing mode, the linewidth is not increased due to this process. But spontaneous emission adds incoherently, without a defined phase relationship to the cavity field and has an inherent Lorentzian distribution of frequencies that yield a finite linewidth. Therefore, spontaneous emission sets a fundamentally lower limit on the laser linewidth [7].



Figure 1: The fundamental lasing transitions absorption, stimulated emission and spontaneous emission.  $E_1$  and  $E_2$  are the energies of lower and upper states, respectively,  $\hbar$  is the reduced Planck constant and  $\omega_0$  the radiation angular frequency. Adapted from [7].

The spontaneous emission linewidth  $\Delta \nu_0$  was calculated by Schawlow and Townes and is given by [7]:

$$\Delta \nu_0 = \frac{\pi \hbar \omega_0 (\Delta \nu_c)^2}{P_{\text{out}}} , \qquad (1)$$

where  $\omega_0$  is the laser (angular-) frequency and  $\Delta \nu_c$  the bandwidth of the used cavity. The dependence on the output power  $P_{\text{out}}$  takes into account that with increasing intra-cavity power stimulated emission becomes more and more the dominating process related to spontaneous emission [7].

In case of a laser wavelength of  $\lambda_0 = 800 \text{ nm}$ ,  $\Delta \nu_c = 10 \text{ MHz}$  and  $P_{\text{out}} = 100 \text{ mW}$  a linewidth of  $\Delta \nu_0 = 0.8 \text{ mHz}$  results.

This limit is typically not reached, since there are various broadening mechanisms which are discussed later on [8]. First, a definition of the linewidth is given. In figure (2) a Lorentzian shaped power spectral density as an example of a potential laser lineshape is shown. Now, the linewidth is some certain width of the distribution. Typically, the full-width at half-maximum (FWHM) is used as shown in the figure indicated by the blue line [1].

Beside the shown Lorentzian lineshape other lineshapes are possible, e.g. described by a Gaussian or a Voigt profile (=convolution of Lorentzian and Gaussian). Note, that for the same linewidth different spectral densities can occur, as shown in figure (3).





Figure 2: Definition of the laser linewidth. As linewidth a certain width of the power spectral density distribution is taken. Here, as most common, it is the full-width at half-maximum (FWHM).

Figure 3: Comparison of Lorentzian and Gaussian lineshape. The FWHM linewidth is indicated by the two red lines and is equal for both lineshapes although the individual shapes differ considerably from each other.

The linewidth of a laser is typically increased by various broadening mechanisms. According to chapter (2.4), the absolute frequencies and the widths of the transmission fringes of the used cavity depend fundamentally on the cavity length. Since the cavity essentially determines the frequency of emitted laser radiation, any mechanical and temperature induced noise that influences the cavity length leads to a raise of noise in the laser frequency and therefore yields an increased linewidth [9].

Moreover, in a gas laser where atoms collide with other atoms, ions, free electrons and the walls of the gas container, respectively, the coherent interaction of the lasing mode and the gas atoms of the active medium during a process of absorption or stimulated emission, is interrupted. This leads to phase shifts and therefore to an increase in linewidth. Since every atom of the ensemble is affected in the same way, the linewidth is *homogeneously broadened* resulting in a Lorentzian lineshape. Thereby, the process of spectral broadening by means of collisions is termed as *collisional broadening* [10].

Collisional broadening can also be found in solid-state lasers  $^4$ . Here collisions are caused due to the interaction of the atom with the lattice phonons [10].

Another broadening mechanism arises from atomic motion and is referred to as *Doppler broadening*. Because of the statistical motion of the atoms of the gain medium relative to the propagation direction of the electromagnetic wave, the frequency of the electromagnetic wave (that interacts with the atoms) as seen in the rest frame of an atom is shifted as compared to the frequency of the wave in the laboratory reference frame according to the *Doppler shift*. From the point of view of atom-radiation interaction, this shift is equivalent to a change of the resonance frequency of the atom. Moreover, taking into account the Maxwell-Boltzmann distribution of velocities in a gas, it can be shown that the distribution of transition frequencies of the ensemble is given again by a Gaussian function and therefore leading to a Gaussian lineshape. Since the individual atoms are affected in a different way, *inhomogeneous broadening* occurs with its typical Gaussian lineshape [10].

There are several more broadening mechanisms depending on the type of laser. More information about this topic can be found, e.g. in [10], [7] or [11].

The lineshape due to homogeneous broadening is always Lorentzian and the lineshape by reason of inhomogeneous broadening is always Gaussian, respectively. If two mechanisms contribute to line broadening, the overall lineshape is given by the convolution of the corresponding lineshape functions. It can be shown that thereby the convolution of a Lorentzian line of full-width at half-maximum  $\Delta\nu_{L1}$  with another Lorentzian line of width  $\Delta\nu_{L2}$  again gives a Lorentzian line of width  $\Delta\nu_{L1} + \Delta\nu_{L2}$ . Besides, the convolution of two Gaussian lines of widths  $\Delta\nu_{G1}$  and  $\Delta\nu_{G2}$  also results in a Gaussian line, but this time with a width of  $\sqrt{\Delta\nu_{G1}^2 + \Delta\nu_{G2}^2}$ . Therefore, for any combination of broadening mechanisms, it is usually

 $<sup>^{4}</sup>$ The pump laser of the optical parametric oscillator, that is the tested laser, is a solid-state laser, too. Therefore, its power spectral density and lineshape, respectively, should be given by a Lorentzian function.

possible to reduce the problem to a convolution of a single Lorentzian with a single Gaussian lineshape. This convolution is known as *Voigt profile*. Sometimes one mechanism predominates. Then it is valid, to consider a pure Lorentzian or Gaussian line [7].

In the beginning, as fundamental reason, why there is a finite linewidth, at all, spontaneous emission was mentioned, since the finite linewidth occurs due to the spontaneous transition of an electron from a higher energy level to a lower one. Moreover, the type of the tested laser is an optical parametric oscillator and is described in chapter (2.2) in more detail. Indeed, optical parametric oscillators are not based on transitions between energy levels building the lasing transition as it is the case in all gas-, liquid- or solid-state lasers. Instead, a parametric process is used to amplify the so called *signal* and *idler* beams occuring in this process by means of the conversion of a *pump* laser. The pump laser of the used optical parametric oscillator is a diode-pumped solid-state laser and is therefore affected by the considerations of a finite linewidth due to spontaneous emission. Thus, the pump laser used for this process has a finite linewidth and this in return leads to a finite linewidth of the signal and idler beams of the optical parametric oscillator.

## 2.2 Optical parametric oscillator

An optical parametric oscillator (OPO) is very similar to a laser, since it emits coherent radiation. Moreover, it makes use of a laser resonator, but in difference to a laser it relies on optical gain from parametric amplification in a non-linear crystal rather than from stimulated emission. Besides, an OPO exhibits a threshold for the pump power, below which there is only parametric fluorescence [5].

In the following, an overview over a certain type of OPO, a singly-resonant optical parametric oscillator SR-OPO, that is the type of the laser used for the experiments, is briefly discussed. More information about OPO's in general can be found in [12], [13], [14], [5], [10] and [7] and their respective references.

As already mentioned, the OPO process takes place in a non-linear crystal, where the second order susceptibility  $\chi^{(2)}$  is used for wave-mixing between three waves, the *pump* wave incident on the crystal at (angular-) frequency  $\omega_p$  and the during the OPO process also appearing so called *signal* (frequency  $\omega_s$ ) and *idler* (frequency  $\omega_i$ ) beams [5]. Since the pump wave is converted into the signal and idler waves, which exhibit longer wavelengths, this process is an optical down conversion process [5]. Thereby, the frequencies of pump, signal and idler satisfy the relation [11]:

$$\omega_{\rm p} = \omega_{\rm s} + \omega_{\rm i} , \qquad (2)$$

where the signal and idler beams are initiated by parametric fluorescence and enhanced during the OPO process during propagation through the non-linear crystal while the pump is depleted to amplify the signal and idler beams [5].

Equation (2) allows any combination of frequencies  $\omega_s$  and  $\omega_i$  and represents energy conservation, that can be seen by multiplying the equation by the reduced Planck constant  $\hbar$ . However, this is restricted by momentum conservation, because on the quantum level each pump photon is destructed to generate a signal and an idler photon. Assuming all three waves to propagate in the same direction, this condition reads [5]:

$$\Delta k = k_{\rm p} - k_{\rm s} - k_{\rm i} \stackrel{!}{=} 0 , \qquad (3)$$

with the wave vectors of pump  $k_{\rm p}$ , signal  $k_{\rm s}$  and idler  $k_{\rm i}$ , respectively.  $\Delta k$  is the phase-mismatch and can be forced to be zero if some phase-matching technique, e.g. quasi-phase-matching (QPM), is used. In this case, as non-linear crystal a periodically poled ferroelectric crystal (e.g. LiNbO<sub>3</sub>) is used, where the sign of the second-order susceptibility coefficient  $\chi^{(2)}$ , that couples the involved waves, is alternated periodically with the modulation period  $\Lambda_{\rm QPM}$  ( $\rightarrow$  periodic poling = periodic change of the  $\chi^{(2)}$  sign) [5]. With an integer *m* and the phase-mismatch in case of QPM  $\Delta k_{\rm OPM}$  equation (3) transforms to:

$$\Delta k_{\rm QPM} = k_{\rm p} - k_{\rm s} - k_{\rm i} - m \frac{2\pi}{\Lambda_{\rm QPM}} = 0 . \qquad (4)$$

This implies, that an additional crystal momentum  $k_{\rm m} = m \frac{2\pi}{\Lambda_{\rm QPM}}$  is utilized to fulfill equation (3) [5]. In addition, the involved pump, signal and idler waves have to propagate in phase to each other as already given by equation (3). If the phase-mismatch  $\Delta k$  does not equal zero, the distance  $L_{\rm coh}$  after that the relative phase between the interacting waves changes by  $\pi$  is constituted by:

$$L_{\rm coh} = \frac{\pi}{\Delta k} \,. \tag{5}$$

After the distance  $L_{\rm coh}$  the generated waves (signal and idler) are back-converted into the pump beam, thus,  $L_{\rm coh}$  is called the coherence length of the OPO process. This is avoided by the already mentioned periodic change of the sign of the second-order susceptibility  $\chi^{(2)}$  with periodicity  $\Lambda_{\rm QPM} = 2L_{\rm coh}$  (m = 1). Therefore, at the point where the conversion with wrong direction would start, the change of sign of  $\chi^{(2)}$ causes an enduring conversion in the desired direction [5].

Using the definition of the wave vector  $k_j = n(T, \omega_j) \frac{\omega_j}{c_0}$  with the refractive index  $n(T, \omega_j)$ , depending on the temperature T and the frequency of the respective wave (j = p, s, i) and the speed of light in vacuum  $c_0$ , equation (4) can be transformed to:

$$n(T,\omega_{\rm p})\frac{\omega_{\rm p}}{c_0} - n(T,\omega_{\rm s})\frac{\omega_{\rm s}}{c_0} - n(T,\omega_{\rm i})\frac{\omega_{\rm i}}{c_0} - m\frac{2\pi}{\Lambda_{\rm QPM}} = 0 , \qquad (6)$$

where  $n(T, \omega_j)$  can be calculated from a suited experimentally determined Sellmeier equation [12]. As a result, the phase-matching condition can be fulfilled by choosing an appropriate temperature T and modulation period  $\Lambda_{\text{QPM}}$  [12].

In addition, the gain of the signal and the idler wave, respectively, fundamentally depends on the phasemismatch  $\Delta k_{\text{QPM}}$  and is highest if  $\Delta k_{\text{QPM}} = 0$  [15].

Considering a small gain, as it is the case in most situations of practical interest, the net fractional gain in signal intensity  $G_s$  under the additional conditions of no pump depletion, zero input idler field and nonzero input signal field is given by:

$$G_{\rm s} = \frac{I_{\rm s}(z=l)}{I_{\rm s}(z=0)} - 1 = \Gamma^2 l^2 \left[\frac{\sin(\Delta k_{\rm QPM} l/2)}{\Delta k_{\rm QPM} l/2}\right]^2 , (7)$$

with z as propagation direction,  $\Gamma$  as gain factor and  $I_{\rm s}(z=0)$  as intensity of the signal beam at the input facet of the non-linear crystal and  $I_{\rm s}(z=l)$  as signal intensity after the propagation length l, respectively [12]. Besides, the part  $\left[\frac{\sin(\Delta k_{\rm QPM}l/2)}{\Delta k_{\rm QPM}l/2}\right]^2 = \operatorname{sinc}^2\left(\frac{\Delta k_{\rm QPM}l}{2}\right)$  of equation (7) is plotted in figure (4). In addition, the full-width at half-maximum of the gain curve FWHM is shown, where the half-maximum is reached if the product of phase-mismatch and propagation length  $\Delta kl$  equals  $\pm 0.44\pi$ .



Figure 4: According to equation (7), the sinc-function part of the net fractional gain of signal intensity  $G_{\rm s}$  is shown.  $\Delta k$  denotes the phase-mismatch in case of QPM  $\Delta k_{\rm QPM}$ . From [10].

As already mentioned in the beginning of this section, as tested laser a singly-resonant OPO is used. For that, consider figure (5) showing the basic layout of such an OPO. It depicts a ring cavity containing the non-linear crystal (NL Crystal) of length  $L_c$  necessary for the OPO-process. The cavity consists of the four mirrors M1, M2, M3 and M4.

Now, in a singly-resonant OPO only one of the signal and idler waves is resonant. Without loss of generality, the signal beam is considered to be resonant here. Then the four mirrors are completely transparent for the pump and idler wave. Besides, three of the mirrors are highly reflective for the signal beam and one mirror (here the M2-mirror) has a slightly reduced reflectivity to couple a small fraction of the signal wave out.

During operation the pump beam passes the non-linear crystal and is partially converted into the signal and idler beams by means of the non-linear interaction. Thereby, the frequency of the idler is determined by energy and momentum conservation (QPM) in combination with the given wavelength of the pump laser and the resonance condition of the cavity.



Figure 5: Layout of an singly-resonant optical parametric oscillator SR-OPO. The four mirrors M1, M2, M3 and M4 realise a ring cavity containing the nonlinear crystal (NL Crystal). In the singly-resonant case only the signal or the idler beam is resonant. Adapted from [5].

#### 2.3 Gaussian beams

An important element of a laser is always the cavity, because it determines the longitudinal and transverse mode structure of the emitted laser radiation and is indicated by the indices m, n (transverse modes) and q (longitudinal mode)<sup>5</sup> [10]. The mode structure and electrical field distribution respectively, fundamentally depend on the mirror shapes, whether they are flat, spherical or something else and the distance of the cavity mirrors, since these properties of the cavity decide upon which kind of modes lead to standing waves in the resonator, that is necessary for a stable cavity, where the beam reproduces itself after one round trip [10].

Vice versa, the mode structure given by a certain cavity is exactly the mode structure that is not suppressed if light of this mode structure is coupled in. I.e. a laser beam incident on a cavity operating in  $\text{TEM}_{q00}$  should match the  $\text{TEM}_{q00}$  mode of the cavity if a high transmission is desired. That is an important point in an experiment if light is coupled into a cavity based interferometer, e.g. a Fabry-Pérot interferometer.

Thereby, the mode structure generally found in cavities is strongly connected with Gaussian beams, since these are eigensolutions of the commonly used cavity structures describing differential equations.

For various reasons most cavities are built to support Gaussian beam operation, especially the fundamental mode. Thus, this mode is discussed in more detail [10].

The electric field of a monochromatic and uniformly polarized light wave at a small angle along the z-direction of a cartesian system of coordinates can be described as:

$$E(x, y, z, t) = E_0 u(x, y, z) e^{i(\omega_0 t - k_0 z)} , \qquad (8)$$

with the light frequency  $\omega_0$ , the wavenumber  $k_0 = \frac{2\pi}{\lambda_0}$ , the optical wavelength  $\lambda_0$  and the electrical field amplitude  $E_0$  [10]. Moreover, u(x, y, z) is the complex field envelope and given by:

$$u(x, y, z) = \frac{\omega_{\rm bw}}{\omega(z)} e^{-\frac{x^2 + y^2}{\omega^2(z)}} e^{-ik_0 \frac{x^2 + y^2}{2R_{\rm beam}(z)}} e^{i\varphi_{\rm G}(z)} , \qquad (9)$$

where  $\omega_{\rm bw}$  is the lowest beam radius the laser beam shows overall. Therefore, it is called the beam waist and located at the position z = 0 by convention. In addition,  $\omega(z)$  plotted in figure (6) is the beam radius for that the electrical field amplitude has decayed by the factor  $\frac{1}{e}$  with respect to the maximum value at the z-axis, which leads a decay in intensity down to  $\frac{1}{e^2}$ . The beam radius  $\omega(z)$  is given by:

$$\omega(z) = \omega_{\rm bw} \sqrt{1 + \left(\frac{z}{z_{\rm R}}\right)^2} , \qquad (10)$$

 $<sup>{}^{5}</sup>$ Of course, light is a transverse wave. Here, longitudinal modes refer to certain modes along the optical axis of the cavity represented by their number of nodes in this direction. In addition, the transverse modes refer to the number of nodes perpendicular to the optical axis.

with the Rayleigh length  $z_{\rm R}$  that is the propagation length after that the beam radius has increased by factor of  $\sqrt{2}$  and the area by 2, respectively. In equation (9)  $R_{\rm beam}(z)$  is the radius of the curvature of the spherical constant-phase wave fronts at the z-axis:

$$R_{\text{beam}}(z) = z \left[ 1 + \left(\frac{z_{\text{R}}}{z}\right)^2 \right] \,. \tag{11}$$

It shows plane-wave like behaviour with flat wave fronts at beam waist  $(R_{\text{beam}}(z \to 0) \to \infty)$ , curved wave fronts everywhere else and is plotted in figure (7). In addition,  $\varphi_{\text{G}}(z)$  is the Gouy-phase, that is an additional phase the Gaussian beam accumulates by propagating along z-axis with respect to a plane-wave [10].

$$\varphi_{\rm G}(z) = \arctan\left(\frac{z}{z_{\rm R}}\right)$$
(12)

Moreover, using equation (10) and

$$z_{\rm R} = \frac{\pi \omega_{\rm bw}^2}{\lambda_0} \tag{13}$$

the (half-angle) beam divergence can be defined [10]:

$$\Theta_{\rm d} = \lim_{z \to \infty} \frac{\omega(z)}{z} = \frac{\lambda_0}{\pi \omega_{\rm bw}} \,. \tag{14}$$

Thus, the function  $f(z) = \Theta_d z$  is the aymptote of the beam radius  $\omega(z)$  for high values of z.



Figure 6: Ratio of beam radius  $\omega(z)$  to the beam waist radius  $\omega_{\rm bw}$  depending on the propagation length z normalised to the Rayleigh length  $z_{\rm R}$ .



Figure 7: Ratio of radius of curvature  $R_{\text{beam}}(z)$ of the constant-phase wave fronts to the Rayleigh length  $z_{\text{R}}$  depending on the propagation length z normalised to the Rayleigh length  $z_{\text{R}}$ .

A Gaussian beam, represented by its beam radius, beam divergence asymptotes and constant-phase wave fronts, is depicted in figure (8).



Figure 8: Beam radius, beam divergence asymptotes and constant-phase wave fronts of a Gaussian beam.

Thereby, the representation of the fundamental mode of a Gaussian beam, given by the equations (8), (9), (10), (11) and (12), can be derived by solving the paraxial Helmholtz equation [10]:

$$\frac{\partial^2 u(x,y,z)}{\partial x^2} + \frac{\partial^2 u(x,y,z)}{\partial y^2} - 2ik_0 \frac{\partial u(x,y,z)}{\partial z} = 0.$$
(15)

A general solution for rectangular boundary conditions, as up to now considered, can be written as in equation (8) stated, but with the mode dependent complex field envelope  $u_{m,n}(x, y, z)$ :

$$u_{\rm m,n}(x,y,z) = \frac{\omega_{\rm bw}}{\omega(z)} H_{\rm m}\left(\frac{\sqrt{2}x}{\omega(z)}\right) H_{\rm n}\left(\frac{\sqrt{2}y}{\omega(z)}\right) e^{-\frac{x^2+y^2}{\omega^2(z)}} e^{-ik_0 \frac{x^2+y^2}{2R_{\rm beam}(z)}} e^{i(1+m+n)\varphi_{\rm G}(z)} , \tag{16}$$

where  $H(X)_i$  are Hermite polynomials of order i. Therefore, setting m = n = 0 would result in the by equation (9) described envelope of the TEM<sub>00</sub> mode. An overview over these so called Hermite-Gaussian modes is shown in figure (9). Moreover, the associated intensity distributions are depicted in figure (10).



Figure 9: Overview of the electric field distribution of Hermite-Gaussian  $\text{TEM}_{\text{mn}}$  modes. Thereby, positive values of the electric field are coloured red, negative ones blue and an electric field of zero is indicated by a green colour. With increase of the positive values, they are deeper red and with decrease of negative values of the electric field, they are deeper blue coloured, respectively. From [16].

As already mentioned, solving the paraxial Helmholtz equation for a rectangular symmetry, i.e. for rectangular, spherical mirrors in a cavity yields Hermite-Gaussian modes. Moreover, choosing a different kind of boundary conditions, would lead to a different set of eigenfunctions. I.e. solving the paraxial wave equation in case of a radial symmetry if circular and spherical mirrors are used, yields Gauss-Laguerre modes, whose intensity distributions are shown in figure (11).

In general any arbitrary field distribution  $E_{\rm arb}(x, y, z, t)$  can be written as linear combination of Hermite-Gaussian modes, since they are a complete set of eigensolutions of equation (15) [16][10].

$$E_{\rm arb}(x, y, z, t) = \sum_{\rm m,n} \left( E_0 u_{\rm m,n}(x, y, z) e^{i(\omega_0 t - k_0 z)} \right)$$
(17)



Figure 10: Overview of the intensity distribution of Hermite-Gaussian  $\text{TEM}_{\text{mn}}$  modes. Thereby, the intensity increases from light blue to deep red. In addition, the individual modes have the same beam waist  $\omega_{\text{bw}}$  and are normalised to have the same power. From [16].



Figure 11: Overview of the intensity distribution of Gauss-Laguerre  $LG_{mn}$  modes. Thereby, the intensity increases from light blue to deep red. In addition, the individual modes have the same beam waist  $\omega_{bw}$  and are normalized to have the same power. From [16].

Moreover, with respect to the fundamental mode higher order modes do not focus as tightly, are more divergent and take power away from the 00-mode [16].

In addition, it should be outlined that just like the cavity mode structure determines the mode structure of emitted laser radiation, it also plays an important role if a Gaussian beam with any kind of mode structure shall be coupled into the cavity. Then, the fraction of light coupled in depends fundamentally on the fact whether the modes match or not. This is called *mode-matching* and typically realised by some suitable optics that transforms the beam that way the beam matches a mode in the cavity. This is discussed further on in chapter (2.4).

In an experiment from the observation of these modes it can be obtained if the cavity is well aligned and if suitable mode-matching optics is used.

#### 2.4 Fabry-Pérot interferometer

A Fabry-Pérot interferometer is the assembly of two opposite mirrors with an alterable spacing between them, i.e. an adjustable optical cavity. To understand the principle, consider the following picture of an optical cavity that has been filled with light. In a classical approach the light travels forth and back along the optical path. At each round-trip a fraction of the stored light is lost because of transmission, scattering, diffraction and absorption at the cavity mirrors. Following this simplified picture, the light will decay in steps determined by the fractional round-trip losses separated in time by the round-trip time. Assuming high mirror reflectivities in addition, the steps are small. The loss is proportional to the stored light which yields an exponential decay. Although here a classical, simplified picture is used, this is exactly the outcome as shown in the following [17].

Now, a mathematically treatment is done to calculate the Fabry-Pérot interferometer describing quantities.

First, the  $n^{\text{th}}$  cavity mirror itself is described by the intensity reflection  $R_n$ , loss  $S_n$  (non recoverable energy losses from scattering, diffraction and absorption) and transmission  $T_n$ , respectively. According to energy conservation, that leads to the relation:

$$R_{\rm n} + S_{\rm n} + T_{\rm n} = 1 . (18)$$

The total losses of the  $n^{\text{th}}$  cavity mirror  $L_{n}$  are given by  $L_{n} = S_{n} + T_{n}$ .



Figure 12: Optical cavity filled with light.  $E_i(t)$  and  $E_o(t)$  are the input and the output electrical field, respectively. Since for this picture a resonant cavity is assumed, no light is reflected back.

Lets now consider light  $E_i(t) = E_i \exp(-i\omega_0 t)$  at angular frequency  $\omega_0$  incident on one mirror [illustrated in figure (12)], which yields the output field  $E_o(t)$  to be [17]:

$$E_{\rm o}(t) = E_{\rm o} \mathrm{e}^{-i\omega_0 t} = E_{\rm i} \mathrm{e}^{-i\omega_0 t} \left[ C_{\rm mnq} \sqrt{T_{\rm i} T_{\rm o}} \frac{1}{1 - \sqrt{R_{\rm p}} \mathrm{e}^{i\delta}} \right] \,. \tag{19}$$

 $R_{\rm p}$  is the product of the mirror reflectivities of input and output mirrors ( $R_{\rm p} = R_{\rm i}R_{\rm o}$ ) and  $C_{\rm mnq}$  is a mode-matching coefficient (*m* and *n* refer to the transversal Gaussian modes and *q* to the longitudinal one).  $E_{\rm o}$  is the amplitude of the output electrical field and  $E_{\rm i}$  of the input field, respectively. In addition, the phase factor  $\delta$  is the accumulated optical phase shift of one round-trip. Using additional phases  $\varphi_{\rm m}$  and  $\varphi_{\rm n}$  for higher order transverse modes, the round-trip optical path length  $L_{\rm opt}$  and the vacuum speed of light  $c_0$ , the phase  $\delta$  is given by:

$$\delta = \frac{\omega_0 L_{\text{opt}}}{c_0} + \varphi_{\text{m}} + \varphi_{\text{n}} \,. \tag{20}$$

Up to now, an incident field  $E_i(t)$  with a constant amplitude was considered. Therefore, the effect on the output field in case of shutting down the input field at t = 0 with a decay constant  $\gamma_s$  is discussed in the following. Under this condition, the input electrical field can be written as:

$$E_{i}(t) = E_{i}e^{-i\omega_{0}t} \qquad t < 0,$$
  

$$E_{i}(t) = E_{i}e^{-(\gamma_{s}+i\omega_{0})t} \qquad t > 0.$$
(21)

Using the decay constant of the cavity field:

$$\gamma_{\rm c} = \frac{c_0}{L_{\rm opt}} \frac{1 - \sqrt{R_{\rm p}}}{\sqrt{R_{\rm p}}} , \qquad (22)$$

the deviation between the frequency of the cavity mnq-th eigenmode  $\omega_{mnq}$  and the frequency of incident light  $\omega_0$ :

$$\Delta\omega_{\rm mnq} = \omega_{\rm mnq} - \omega_0 \;, \tag{23}$$

the mode-matching coefficient:

$$A_{\rm mnq} = C_{\rm mnq} \sqrt{\frac{T_{\rm i} T_{\rm o}}{R_{\rm p}}} , \qquad (24)$$

and equations (19) and (21), the time dependent output field  $E_{o}(t)$  reads [17]:

$$E_{\rm o}(t) = E_{\rm i}A_{\rm mnq}\frac{c_0}{L_{\rm opt}}\frac{1}{\gamma_{\rm c} + i\Delta\omega_{\rm mnq}}e^{-i\omega_0 t}$$
  
for  $t < 0$  and  
$$E_{\rm o}(t) = E_{\rm i}A_{\rm mnq}\frac{c_0}{L_{\rm opt}}\left[\frac{1}{\gamma_{\rm c} - \gamma_{\rm s} + i\Delta\omega_{\rm mnq}}e^{-(\gamma_{\rm s} + i\omega_0)t} + \frac{1}{\left(1 - \frac{\gamma_{\rm c}}{\gamma_{\rm s}} - i\frac{\Delta\omega_{\rm mnq}}{\gamma_{\rm s}}\right)(\gamma_{\rm c} + i\Delta\omega_{\rm mnq})}e^{-(\gamma_{\rm c} + i\omega_{\rm mnq})t}\right]$$
  
for  $t > 0.$  (25)

The equation for t < 0 describes the steady state response of the cavity to the input field and the equation for t > 0 represents the response of the cavity to shutting off the input field [17].

In addition, the equation can be simplified if the shut down of the input field is much faster than the decay rate in the cavity ( $\gamma_s >> \gamma_c$ ), which leads for t > 0 to:

$$E_{\rm o}(t) = E_{\rm i} A_{\rm mnq} \frac{c_0}{I_{\rm opt}} \frac{1}{\gamma_{\rm c} + i\Delta\omega_{\rm mnq}} e^{-(\gamma_{\rm c} + i\omega_{\rm mnq})t} .$$
<sup>(26)</sup>

This equation shows that the output electric field decays to zero after shutting down the input field at t = 0. In addition, the output field, unlike the input field, oscillates at the cavity resonance frequency [17].

After determining the intensity I(t) in an experiment, the following equation can be derived from equation (26), which is plotted in figure (13). Thereby, the measurement of the decaying intensity after shutting down the light source leads to a cavity decay time  $\tau_c$  and is called *cavity ring-down measurement*.

$$I(t) = I_0 \mathrm{e}^{-\frac{t}{\tau_{\mathrm{c}}}} \tag{27}$$

Here, the cavity decay time  $\tau_{\rm c}$  and the steady-state intensity  $I_0$  are given by:

$$\tau_{\rm c} = \frac{1}{2\gamma_{\rm c}} ,$$

$$I_0 = \frac{1}{2} c_0 \varepsilon_0 \left| E_{\rm o}(t < 0) \right|^2 ,$$
(28)

with the speed of light  $c_0$  and the vacuum permittivity  $\varepsilon_0$ .

Thus, by measuring the output light's exponential decay after shutting down the laser source, the cavity's decay time can be determined.

Moreover, using equation (22) yields:

$$\frac{\sqrt{R_{\rm p}}}{1 - \sqrt{R_{\rm p}}} = 2\frac{c_0}{L_{\rm opt}}\tau_{\rm c} \ . \tag{29}$$

Thus, if the round-trip optical path length  $L_{\text{opt}}$  and the cavity decay time  $\tau_{\text{c}}$  are measured, the product of the mirror reflectivities  $R_{\text{p}}$  can be calculated.



Figure 13: Cavity ring-down curve. If the input field of the cavity is shut down, the stored light will decay exponentially.

With the optical path length  $L_{opt}$  and the cavity decay time  $\tau_c$  known, the finesse of the cavity can be calculated. The finesse F is defined as the ratio of the cavity free spectral range  $\Delta \nu_{FSR}$  to the intensity response function's full-width at half-maximum  $\Delta \nu_c$  (the FWHM of the cavity transmission fringe in frequency domain) [17]:

$$F = \frac{\Delta\nu_{\rm FSR}}{\Delta\nu_{\rm c}} \,. \tag{30}$$

Moreover, the finesse is given by the mirror reflectivities [17]:

$$F = \frac{\pi R_{\rm p}^{\frac{1}{4}}}{1 - \sqrt{R_{\rm p}}} \quad \left( = \frac{\pi \sqrt{R_{\rm n}}}{1 - R_{\rm n}} \right) , \qquad (31)$$

where for the term in brackets, it was assumed, that the mirror reflectivities of both mirrors are equal  $(R_{\rm i} = R_{\rm o} = R_{\rm n})$ . Besides, using equations (22) and (28) and the approximation  $\sqrt[4]{R_{\rm p}} \approx 1$ , the finesse F can be expressed by the cavity decay time  $\tau_{\rm c}$  and the free spectral range  $\Delta \nu_{\rm FSR}$  or the round-trip optical path length  $L_{\rm opt}$ , respectively:

$$F = 2\pi \frac{c_0}{L_{\text{opt}}} \tau_{\text{c}} = 2\pi \Delta \nu_{\text{FSR}} \tau_{\text{c}} .$$
(32)

Here, the relation

$$\Delta \nu_{\rm FSR} = \frac{c_0}{L_{\rm opt}} \tag{33}$$

was used [17].

In summary, the finesse F is given by the mirror reflectivities and determines the ratio of the cavity free spectral range  $\Delta \nu_{\rm FSR}$  to the cavity bandwidth  $\Delta \nu_{\rm c}$ . Thereby, the free spectral range  $\Delta \nu_{\rm FSR}$  only depends

on the optical round-trip path length  $L_{opt}$  in the cavity. In addition, since the mirror reflectivity (and therefore the finesse) is a fixed quantity, the cavity bandwidth depends directly on the optical round-trip path length  $L_{opt}$ . Moreover, in an experiment the finesse F is determinable by measuring  $L_{opt}$  and the decay time  $\tau_c$ . Then, using equations (31) and (32) yields the finesse and the mirror reflectivities.

Moreover, the transmission spectrum of the Fabry-Pérot interferometer is a quantity of interest, since it represents the frequency filtering properties of it. It can be derived from equation (19) by calculating the ratio of the output intensity to the input intensity and is therefore given by the square of the absolute value of the term in brackets. Assuming a perfect mode-matching ( $C_{mnq} = 1$ ), the cavity transmission spectrum  $T(\delta)$ , depending on the accumulated round-trip phase shift  $\delta$ , can be written as:

$$T(\delta) = \left|\frac{E_{\rm o}}{E_{\rm i}}\right|^2 = \frac{1}{1 + \frac{4\sqrt{R_{\rm p}}}{(1 - \sqrt{R_{\rm p}})^2} \sin^2(\frac{\delta}{2})} \,. \tag{34}$$

With equation (31) follows:

$$T(\delta) = \frac{1}{1 + \frac{4F^2}{\pi^2} \sin^2(\frac{\delta}{2})} .$$
(35)

 $T(\delta)$  has the form of an Airy function and is plotted for two different values of the finesse F in figure (14) whereby  $\delta = \frac{\omega_0}{\Delta \nu_{\rm FSR}} = 2\pi \frac{\nu_0}{\Delta \nu_{\rm FSR}} = 2\pi \frac{L_{\rm opt}}{\lambda_0}$  with laser frequency  $\nu_0$  and wavelength  $\lambda_0$  was used [18]. The full optical round-trip path length  $L_{\rm opt}$  in the cavity equals twice the mirror spacing  $d_{\rm m}$  (refractive index is taken as one). In addition, the cavity resonance bandwidth  $\Delta \nu_{\rm c}$ , that is the full-width at half-maximum of the cavity transmission fringe and the cavity free spectral range  $\Delta \nu_{\rm FSR}$  are shown. Here it becomes clear that with an increasing finesse (increasing mirror reflectivity) the bandwidth decreases and, therefore, a higher resolution can be obtained.

This is taken into account by the resolving power  $R_{\text{FPI}}$  of a Fabry-Pérot interferometer and is given by [19][11]:

$$R_{\rm FPI} := \frac{\lambda_0}{\Delta \lambda_0} = \frac{\tilde{\nu}_0}{\Delta \tilde{\nu}_0} = qF .$$
(36)

Here,  $\lambda_0$  is the wavelength and  $\Delta\lambda_0$  is the minimal deviation that can be resolved referring to the Rayleigh criterion. This can be expressed by frequencies using  $\tilde{\nu}_0$  as the operating frequency and  $\Delta\tilde{\nu}_0$  as the minimal resolvable deviation from  $\tilde{\nu}_0$ . In addition, the mode number q is determined by:

$$q = \frac{L_{\text{opt}}}{\lambda_0} . \tag{37}$$

It follows, that in order to achieve a high resolving power, a long, high finesse cavity (with high mirror reflectance) is needed. E.g. for  $\lambda_0 = 1260 \text{ nm}$ ,  $L_{\text{opt}} = 70 \text{ cm}$  and R = 0.99995 a resolving power of  $R_{\text{FPI}} = 3.5 \cdot 10^{10}$  is achievable! Thus, from equations (36) and (37) the necessary values of R and  $L_{\text{opt}}$  in order to achieve a certain resolution in an experiment is obtainable.

In a real experiment, the resolving power is reduced due to imperfections of the mirrors (e.g. bad surface flatness) and mirror alignment, but these influences are strongly reduced if concave mirrors instead of plane ones are used [20].



Figure 14: Transmission spectrum of a Fabry-Pérot interferometer. Although mirrors with high reflectivity are considered, in steady state condition at resonance frequency the transmission can achieve 100% (in case of no diffraction and absorption losses).

The resonances shown in figure (14) only consider the fundamental Gaussian modes (m = n = 0). For higher order transverse modes, according to equation (20), an additional round-trip phase that shifts the resonance frequencies is given. The frequency shift  $\Delta \nu_{\rm mn}$  relative to the frequency of the fundamental mode can be expressed by [21][22]:

$$\Delta \nu_{\rm mn} = (m+n) \Delta \nu_{\rm FSR} \frac{\varphi_{\rm G,rt}}{2\pi} .$$
(38)

Here,  $\varphi_{G,rt}$  is the phase shift per round-trip of the Gouy phase in the optical cavity and is stated by [10]:

$$\varphi_{\rm G,rt} = 2 \cdot \arccos\left\{ \sqrt{\left(1 - \frac{d_{\rm m}}{R_{\rm rad,1}}\right) \left(1 - \frac{d_{\rm m}}{R_{\rm rad,2}}\right)} \right\} , \qquad (39)$$

where  $d_{\rm m}$  is the mirror spacing and  $R_{\rm rad,n}$  is the radius of curvature of the n<sup>th</sup> mirror, respectively. The origin of  $\Delta\nu_{\rm mn}$  can be found in equation (16) in the term exp  $[i(1 + m + n)\varphi_{\rm G}(z)]$ , which leads to an additional phase shift during propagation compared to the fundamental mode. The fundamental and higher order modes in frequency domain are illustrated in figure (15) and can be calculated by simply considering the cavity geometry as given by the equations (38) and (39).



Figure 15: Fundamental and higher order Gaussian modes in the frequency domain. Thereby, the frequencies of the higher order transverse modes relative to the fundamental longitudinal modes is given by the geometry of the cavity.

Up to now, incident light that matches a resonance of a cavity with a fixed optical path length was considered. This condition is dropped, now. If the spacing between the cavity mirrors is changed with a velocity v, the frequency of a certain cavity resonance  $\nu_{c,j}$  changes due to the relation:

$$\nu_{\rm c,j} = j \cdot \Delta \nu_{\rm FSR} = j \cdot \frac{c_0}{L_{\rm opt}} , \qquad (40)$$

with j as an integer. Moreover, the frequency of the incident laser light is fixed. So if initially the laser frequency does not match a resonance of the cavity, nearly no light is transmitted until the length is changed in a way that the cavity transmission peak and the laser line shape overlap in frequency domain again, which is illustrated in figure (16). Here, the cavity transmission fringe moves with an increase of the optical path length towards longer wavelengths because of equation (40) and the relation between wavelength and frequency  $\lambda_{c,j} = \frac{c_0}{\nu_{c,i}}$ .



Figure 16: Effect of increasing the cavity length on a certain transmission fringe represented by transmission fringes shown for three different cavity lengths. In addition, a in wavelength domain fixed Gaussian laser line shape is shown, too.

The time of this overlap is proportional to the convolution of laser line shape and transmission peak and the scan speed. Thus, changing the mirror spacing with a higher velocity v leads to a lower level of transmitted intensity since there is less time to built up the cavity field.

The ratio of transmitted to incident light intensity is called the magnitude of the fringe MF and is given by [6]:

$$MF(l) = \frac{I_{\text{out}}(l)}{I_0} = T^2 \int_0^\infty \mathrm{d}\nu \frac{1}{\pi} \frac{\frac{\Delta\nu_0}{2}}{(\nu - \nu_0)^2 + (\frac{\Delta\nu_0}{2})^2} \cdot R^{2[l+\delta(\nu)]} \cdot \left| \sum_{n=-[l+\delta(\nu)]}^\infty R^n \exp\left\{i2\pi\nu t_r \frac{v}{c_0} n^2\right\} \right|^2 \ . \ (41)$$

A detailed derivation can be found in [23], [24] and [25]. Here, T is the (intensity-) transmittance and R the (intensity-) reflectance of the cavity mirrors. For the calculation both mirrors are assumed to be equal. Therefore, according to equation (18),  $T = T_i = T_o$  and  $R = R_i = R_o = \sqrt{R_p}$  hold. Besides,  $\Delta \nu_0$  and  $\nu_0$  are the laser linewidth and carrier frequency, v is the velocity of the moving mirror and  $t_r$  is the cavity round trip time ( $= L_{opt}c_0^{-1}$ ). Furthermore, l is the time t normalized to the round-trip time  $t_r$  and is therefore dimensionless:

$$l = \frac{t}{t_{\rm r}} \,. \tag{42}$$

In addition,  $\delta(\nu)$  is the dimensionless time difference between resonant instances for the frequencies  $\nu$  and  $\nu_0$  [6]:

$$\delta(\nu) = -\frac{c_0(\nu - \nu_0)}{2\nu\nu_0} \,. \tag{43}$$

Furthermore, the dependence of the exponential function's phase  $\pi \nu t_r \frac{2v}{c_0} n^2$  on the mirror velocity v accounts for the doppler shift that is introduced with each reflection of light at the moving mirror and the term:

$$\frac{1}{\pi} \frac{\frac{\Delta \nu_0}{2}}{(\nu - \nu_0)^2 + (\frac{\Delta \nu_0}{2})^2} =: g_{\rm L}(\nu) \tag{44}$$

represents the Lorentzian lineshape of the tested laser. In case of another lineshape it is to be replaced. E.g. for a Gaussian lineshape it has to be exchanged by [8]:

$$g_{\rm G}(\nu) := \frac{2}{\Delta\nu_0} \left(\frac{\ln(2)}{\pi}\right)^{\frac{1}{2}} \exp\left\{-\frac{\ln(2)(\nu-\nu_0)^2}{(\frac{\Delta\nu_0}{2})^2}\right\} \,. \tag{45}$$

Moreover, in figure (17) equation (41) is plotted for the three different laser linewidths  $\Delta\nu_0 = 10 \,\mathrm{kHz}$ ,  $\Delta\nu_0 = 100 \,\mathrm{kHz}$  and  $\Delta\nu_0 = 500 \,\mathrm{kHz}$  against the scan velocity. Thereby, an optical pathlength  $L_{\rm opt} = 68 \,\mathrm{cm} = 2 \cdot d_{\rm m} = 2 \cdot 34 \,\mathrm{cm}$  with the mirror spacing  $d_{\rm m}$  was assumed (refractive index = 1) and the relations T = 1 - R and  $\tau_{\rm c} = \frac{t_{\rm r}}{-\ln(R^2)}$  [26] were used. In addition, the scan velocity is denoted in *free* spectral ranges per time. Since, the transmission spectrum of the Fabry-Pérot interferometer repeats itself [see figure (14)] and the repetition is given after a change in mirror spacing  $d_{\rm m}$  by  $\Delta d_{\rm m} = \frac{\lambda_0}{2}$ , the scan of one free spectral range corresponds to this change in length. Thus, the following relation can be derived:

$$n_{\rm FSR} \,[{\rm FSR}] = \frac{\Delta d_{\rm m}}{\frac{\lambda_0}{2}} \,. \tag{46}$$

Here,  $n_{\text{FSR}}$  in units of free spectral ranges states how often the transmission spectrum is repeated for a given length change  $\Delta d_{\text{m}}$ . Furthermore, using the mirror velocity  $v = \frac{\Delta d_{\text{m}}}{t}$  and  $\tilde{n}_{\text{FSR}}\left[\frac{\text{FSR}}{\text{s}}\right] := \frac{n_{\text{FSR}}[\text{FSR}]}{t}$ , a relation between the mirror velocity v and the number of scanned free spectral ranges per second,  $\tilde{n}_{\text{FSR}}\left[\frac{\text{FSR}}{\text{s}}\right]$  can be calculated:

$$\tilde{n}_{\rm FSR} \left[ \frac{\rm FSR}{\rm s} \right] = \frac{2v}{\lambda_0} \,. \tag{47}$$



## Magnitude of the fringe depending on the laser's linewidth

Figure 17: Normalized magnitude of the fringe for three different linewidths  $\Delta \nu_0 = 10 \text{ kHz}$ ,  $\Delta \nu_0 = 100 \text{ kHz}$  and  $\Delta \nu_0 = 500 \text{ kHz}$ . Thereby, each MF curve is normalized to its maximum.

If the stability of the cavity is considered or light is coupled into the cavity (referring to mode-matching), according to chapter (2.3), the transverse mode structure has to be taken into account. Thereby, the cavity stability fundamentally depends on the mirror spacing  $d_{\rm m}$  and the radius of curvature of the individually

mirrors  $R_n$  [22]. In figure (18) a fundamental Gaussian mode represented by its beam radius and curved constant-phase wave fronts in a cavity is shown. The cavity consists of the spherically mirrors  $M_1$  and  $M_2$  separated in distance by  $d_m$ .



Constant phase front; it's radius of curvature matches that of the mirror but only at the mirror.

Figure 18: Fundamental Gaussian mode represented by its beam radius (dashed curved lines) in a cavity with the spherically mirrors  $M_1$  and  $M_2$  in a distance  $d_m$ . In addition, the curved constant-phase wave fronts (continuous lines) are shown. From [16].

To achieve a stable cavity, where the beam reproduces itself after one round-trip, the following condition has to be fulfilled [10][22]:

$$0 < \left(1 - \frac{d_{\rm m}}{R_{\rm rad,1}}\right) \left(1 - \frac{d_{\rm m}}{R_{\rm rad,2}}\right) < 1 , \qquad (48)$$

and is, therefore, an important design criterion for building a Fabry-Pérot interferometer. Here,  $R_{\rm rad,n}$  is the radius of curvature of the n<sup>th</sup> mirror.

Since, referring to chapter (2.3), the mode structure of the incident beam has to match the mode structure of the cavity to couple as much light as possible into it, some knowledge about the cavity modes is necessary. If a  $\text{TEM}_{00}$  mode is assumed for the incoming beam, the fundamental cavity mode is to be analysed.

The radius of curvature  $R_{\text{beam}}$  [given by equation (11)] of the constant-phase wave fronts of the circulating beam in the cavity has to match the radius of curvature of the mirrors  $R_{\text{rad,n}}$  at the mirrors [16]. Assuming  $z_1$  and  $z_2$  to be the positions of the first and second mirror, respectively, this condition reads:

$$R_{\text{beam}}(z_1) = -R_{\text{rad},1}$$
 and  $R_{\text{beam}}(z_2) = R_{\text{rad},2}$ , (49)

with  $z_2 - z_1 = d_m$ . From this the Rayleigh range of the beam in the cavity  $z_R$  can be calculated [16].

$$z_{\rm R}^2 = \frac{d_{\rm m}(R_1 - d_{\rm m})(R_2 - d_{\rm m})(R_1 + R_2 - d_{\rm m})}{(R_1 + R_2 - 2d_{\rm m})^2}$$
(50)

Moreover, equation (13) yields the beam waist radius  $\omega_{\rm bw}$  in the cavity.

$$\omega_{\rm bw} = \sqrt{\frac{\lambda_0 z_{\rm R}}{\pi}} \tag{51}$$

E.g. in case of a cavity with  $d_{\rm m} = 34$  cm, equal mirrors with  $R_1 = R_2 = 1$  m and a wavelength of  $\lambda_0 = 1000$  nm one gets out a Rayleigh range of  $z_{\rm R} = 37.6$  cm and a beam waist radius of  $\omega_{\rm bw} = 346 \,\mu$ m. In addition, in figure (19) the spotsizes of the fundamental cavity mode dependent on the wavelength are shown. Thereby, the spotsize is given by two times the beam waist radius  $\omega_{\rm bw}$ .



Mode-matching: Spotsize of fundamental cavity mode at beam waist

Figure 19: Mode-matching: Spotsize of the fundamental cavity mode at beam waist. Calculated for  $d_{\rm m} = 38$  cm and  $R_1 = R_2 = 1$  m.

Because an incident beam probably would has different values of Rayleigh range and beam waist after passing the first cavity mirror, some optics to transform and adjust the incident Gaussian beam has to be used. In figure (20) a suitable mode-matching optics consisting of two plano-convex lenses (a beam telescope) is shown.

If the focal length of the first lens  $f_1$  is larger than the one of the second lens  $f_2$  the beam radius is decreased by the ratio of these focal lengths. In general, the magnification  $M_{\text{mag}}$  of this type of beam telescope is given by [27]:

$$M_{\rm mag} = \frac{f_2}{f_1} \,.$$
 (52)

In addition, the location where the beam telescope is placed and the distance between the lenses determine the location of the new beam waist after being transformed by the telescope.

In an experiment the laser beam incident on the cavity (without any mode-matching optics in the path) can be analysed with a beam propagation analyser to determine the initial beam parameters regarding the position and size of the beam waist respectively, the Rayleigh length and the beam quality parameter  $M^2$ . Then, a software using the *ABCD-matrix method for Gaussian beams* can be used in order to calculate the best positions of the lenses to obtain a good mode-matching between the incident beam after passing the first cavity mirror and the fundamental cavity mode<sup>6</sup>.

In addition, in combination with the beam telescope an aperture can be used to avoid higher order transverse modes of the incident beam, since they exhibit different resonance frequencies.

 $<sup>^{6}</sup>$ Within this thesis the freeware *reZonator* was used to do these calculations.



Figure 20: Mode-matching optics to adjust the incident beam to the fundamental cavity mode. Although, plano-convex lenses are typically placed in the beam with the curved front in direction towards the incident beam, here, the first lens is reversed to avoid the realisation of another Fabry-Pérot cavity, that could influence the incident laser beam. The aperture (indicated by the black lines) is used in order to avoid higher order transverse modes.

## 2.5 Acousto-optic modulator

Since an acousto-optic modulator is of high importance in several parts of the experiments, it is discussed in more detail.

An acousto-optic modulator (AOM) is a device used to control the power, frequency or spatial direction of a laser beam with an electrical drive signal [28].

It consists of a crystal that is transparent for the used wavelength and a piezo electric transducer attached to this crystal to excite a sound wave of frequency  $\nu_{AOM}$  in it and build up a periodic modification of the refractive index by the oscillating mechanical pressure of the sound wave based on the acousto-optic effect. Moreover, the introduced periodic modification is equal to a diffraction grating and the laser beam propagates through the crystal. This leads to Bragg diffraction of the light, but in difference, the diffracted light is frequency shifted by (positive and negative) integer multiples of the sound wave frequency because of the momentum the sound wave exhibits and propagates in a slightly different direction [28]. Thereby, the increase of the angle of change in direction is proportional to the frequency shift and is shown in figure (21).



Figure 21: Principle of an acousto-optic modulator. The laser beam propagating through the crystal (frequency  $\nu_0$ ) is diffracted due to a grating introduced by a sound wave and frequency shifted by (positive and negative) integer multiples of the soundwave frequency  $\nu_{AOM}$ .

The distance of the periodically repeated planes of equal density (and therefore of equal refractive index)  $\Lambda$  is given by:

$$\Lambda = \frac{v_{\rm s}}{\nu_{\rm AOM}} , \qquad (53)$$

with  $v_s$  as the velocity of the sound wave in the crystal [10]. Moreover, using Bragg's law, the angles of diffraction  $\Theta_n$  for that the condition of constructive interference is fulfilled in this periodic structure can

be calculated by:

$$\sin(\Theta_{\rm n}) = n \frac{\lambda_0}{2\Lambda} , \qquad (54)$$

with  $\lambda_0$  as the wavelength of the laser beam in the medium and n as the diffraction order [10]. The frequency shift of the laser beam depends directly on the diffraction order n [9]:

$$\nu_{0,n} = \nu_0 + n \cdot \nu_{AOM} \,. \tag{55}$$

with the laser frequency before diffraction  $\nu_0$  and after diffraction into the order  $n \nu_{0,n}$  (with n = ..., -1, 0, 1, ...). Thereby, the fraction of incident intensity diffracted into a certain order, the diffraction efficiency, is typically in the order of 65 % - 85 % and depends on the used AOM and wavelength, size and divergence angle, respectively, of the modulated laser beam [28].

Furthermore, it is possible to use an AOM as an optical shutter if the light of any order except the zeroth order is used, since only the drive signal of the AOM has to be switched off. Then, the laser beam is turned off within a few tens or hundreds of nanoseconds.

From equations (53) and (54) it follows, that shifting the frequency of the laser beam by  $\nu_{AOM}$  means shifting the beam alignment ( $\Theta_n$ ) simultaneously. This may be a big alignment issue if the frequency shift is changed during an experiment and can be avoided by using an *AOM double-pass configuration* (AOM-DPC), as shown in figure (22) [29][30][31]. Here, a horizontally polarised (w.l.o.g.) laser beam passes a polarising beam splitter (PBS), where horizontally polarised light is transmitted and vertically polarised light is reflected off (indicated by the dashed red line), respectively.

Next, the beam polarisation is turned from linear to circular by means of a quarter-wave plate ( $\lambda/4$ , QWP) and focused into an acousto-optic modulator (AOM) due to a bi-convex lens (BCL). Thereby, the distance between this lens with focal length  $f_{\rm BCL}$ and the AOM equals  $f_{\rm BCL}$ . By this, the beam waist of the laser beam with its small size and divergence is located in the AOM that is best to achieve a high diffraction efficiency (the laser beam has to be smaller than the active aperture of the AOM and has to fulfil the Bragg angle).

After the AOM a plano-convex lens (PCL) with focal length  $f_{PCL}$  is placed, again with a spacing between AOM and lens equal to the focal length of the used lens  $(f_{PCL})$ . Besides, behind the lens all diffraction orders, except the first one, are blocked by an aperture (A) and only the first order is reflected back by means of a tiltable plane mirror (TM), afterwards. Due to this arrangement the first order is always retro reflected into itself, even if the angle of the first order is changed.

Then, the first diffraction order propagates through the AOM a second time, is diffracted again and the new first diffraction order propagates collinearly with the incident beam<sup>a</sup>.

The laser beam passes the QWP again, which leads to vertically polarised light. Therefore, this time the light is reflected at the PBS and can be used in an experiment. By this, the PBS in combination with the QWP distinguishes between light travelling towards the AOM and backwards, respectively.

The most important effect of this double-pass configuration on the beam is the fact, that the alignment of the light outcoupled at the PBS is no longer frequency dependent. Besides, the frequency shift introduced to the light is doubled, because there are always two passes through the AOM.



Figure 22: An acousto-optic modulator in double-pass configuration. The following symbols are used:  $\frac{\lambda}{4}$  (quarter-wave plate), A (aperture), AOM (acousto-optic modulator), BCL (bi-convex lens), PBS (polarising beam splitter), PCL (plano-convex lens) and TM (tiltable mirror). Adapted from [31].

To control the sound wave frequency in the AOM and, therefore, the introduced frequency shift, a voltage controlled oscillator can be used. This device generates an electrical drive signal with a frequency depending nearly linearly on the applied voltage.

## 2.6 Delayed self-heterodyne interferometry

Delayed self-heterodyne interferometry (DSHI) is a suitable technique to measure linewidths down to a few tens of kHz if wavelengths are used, where optical fibres exhibit only low attenuation and an already existing DSHI setup is used as reference for the Fabry-Pérot interferometer setup. Therefore, DSHI is discussed in more detail.

<sup>&</sup>lt;sup>a</sup>The laser beam incident on the AOM in direction BCL-AOM-PCL is diffracted and the first order is used after retro-reflection as new incident beam in direction PCL-AOM-BCL. This beam is diffracted again. Thereby, by this configuration the beam alignment is automatically optimised for the diffraction into the order that is collinear with the incident beam in direction BCL-AOM-PCL.



Figure 23: Principle setup of delayed self-heterodyne interferometry. This figure is described in the text below.

The principle setup is shown in figure (23). The light of the tested laser with carrier frequency  $\nu_s$  and the power spectral density's full-width at half-maximum  $\Delta \nu$  is coupled into an optical fibre [32]. To prevent feedback from the setup, that could influence the lasers output spectrum, an optical isolator is used. Afterwards, the beam is split into two paths. On the first one, a frequency shift  $\delta \nu$  is introduced to the laser light by an acousto-optic modulator (AOM) and leads to an oscillation frequency of  $\nu_{\rm LO} = \nu_{\rm s} - \delta \nu$ . The second path serves as delay line of length  $L_{\rm delay}$  with length slightly longer than the coherence length of the tested laser. Then, the two paths are combined again. Therefore, the frequency shifted light of the laser is superimposed with a not frequency shifted but delayed version of it. Hence, a beat note of two quasi independent lasers (one path is longer than the coherence length) occurs which is detected by a photo diode (PD). Here, the incident intensity I(t) is given by [32]:

$$I(t) = P_{\rm s}(t) + P_{\rm LO}(t) + \sqrt{P_{\rm s}(t)P_{\rm LO}(t)}\cos[2\pi(\nu_{\rm s} - \nu_{\rm LO})t + \Delta\varphi(t)],$$
(56)

where  $P_{\rm s}(t)$  is the power of the delayed version of the beam and  $P_{\rm LO}(t)$  the power of the frequency shifted one, respectively. The third term indicates the interference beat note. It is modulated with a cosine function, whereby the cosines frequency depends on the introduced frequency shift  $\delta \nu = \nu_{\rm s} - \nu_{\rm LO}$ and the instantaneous phase difference  $\Delta \varphi(t)$  between the two interfering beams. The output current of the PD is analysed with an electrical spectrum analyser (ESA).

In figure (24) the power spectral densities  $S_{\rm E}(\nu)$  of the delayed laser beam (top) and the frequency shifted version (middle) are shown. Since both beams exhibit the same line shape and carrier frequency  $\nu_{\rm s}$ , if they are split, and then the frequency of one beam is shifted by  $\delta\nu$ , both beams show the same power spectral density (PSD) but shifted in frequency domain by  $\delta\nu$ . In addition, the PSD  $S_{\rm i}(\nu)$  of the interference beat note, obtained with the ESA, is depicted. It has twice the linewidth of the tested laser and is centred in frequency at  $\delta\nu$ . From that, the line width of the tested laser can be derived by fitting a convolution of Gaussian and Lorentzian function to the data [32].



Figure 24: Power spectral densities  $S_{\rm E}(\nu)$  of the delayed laser beam (top) and the frequency shifted version (middle). Besides, the power spectral density  $S_{\rm i}(\nu)$  of the interference beat note, detected by the PD and analysed with the ESA, is shown (bottom). Thereby,  $\Delta\nu$  is the tested laser's linewidth centred around its carrier frequency  $\nu_{\rm s}$  and  $\delta\nu$  is the frequency shift introduced by the AOM [32].

If one calculates the introduced time delay due to the optical fibre  $\tau_{\text{delay}} = \frac{L_{\text{delay}}}{c_{\text{fibre}}}$  with the speed of light in the fibre  $c_{\text{fibre}}$ , the resolution  $R_{\text{DSHI}}$  of delayed self-heterodyne interferometry is given by [33]:

$$R_{\rm DSHI} = \frac{1}{\tau_{\rm delay}} \,. \tag{57}$$

E.g. for  $L_{\rm delay} = 25 \,\rm km$  and a refractive index  $n_{\rm fibre} = 1.4511$  a resolution of  $R_{\rm DSHI} \approx 8.3 \,\rm kHz$  can be obtained but the laser and the setup have to be completely stable over the whole delay time that is  $\tau_{\rm delay} = 121 \,\mu s$  in this case.

## 3 Linewidth measurement with a Fabry-Pérot interferometer using a length modulation technique

## 3.1 Experimental setup and performance

#### 3.1.1 Cavity characterisation measurement

This setup is used to characterise the cavity (the Fabry-Pérot interferometer), which is necessary for the linewidth measurement afterwards and is presented in figure (25). According to equation (32) it serves the purpose of determining the cavity decay time  $\tau_c$  and the free spectral range  $\Delta \nu_{\rm FSR}$  to calculate the finesse F of the cavity and the reflectivities R of its mirrors.

In the following, the mirrors used in the setup are presented in figure (25) (indicated by M and TM)<sup>7</sup> but are not mentioned in the text which is describing the figure (25).

First, the linearly polarised light of the tested laser incidences on a half-wave plate  $(\lambda/2)$  to adjust the polarisation to be exactly horizontal in order to achieve high transmission at a polarising beam splitter (PBS) afterwards. By this, a high fraction of the linearly polarised laser light is usable for the *AOM double-pass configuration* (AOM-DPC), that is discussed in detail in chapter (2.5). In addition, the laser beam is slightly focussed by a bi-convex lens (BCL1, focal length = 400 mm).

At the PBS, the first element of the AOM-DPC, only horizontally polarised light is transmitted and vertically polarised light is reflected off (indicated by the dashed red line) and absorbed by a beam dump (BD). The transmitted horizontally polarised light is turned into circularly polarised light by means of a quarter-wave plate ( $\lambda/4$ , QWP) and focussed into an acousto-optic modulator (AOM, *Brimrose TEF-200-50-900/1300*, driven by a *Brimrose VFF-200-50-V-B1-V2-ARF* voltage controlled oscillator VCO) by a bi-convex lens (BCL2, focal length = 75 mm). Then, light of the first diffraction order is retro-reflected into itself by a combination of plano-convex lens (PCL1, Focal length = 75 mm), aperture (A) and mirror (M) and diffracted at the AOM a second time. Thereby, the distances between the BCL2 and the AOM and the PCL1 and the AOM, respectively, are given by their focal lengths  $f_{BCL2}$  and  $f_{PCL1}$ . The laser light now frequency shifted twice, is turned into vertically polarised light due to the second pass through the QWP and split off at the PBS towards the cavity. Therefore, the combination of PBS and QWP serves as an optical isolator to avoid any back-reflection into the tested laser and is necessary for the used AOM-DPC.

Moreover, the beam alignment of the AOM is no longer frequency dependent, which is necessary for a good coupling into the cavity regarding mode-matching, later.

After the PBS, there is a second combination of a PBS and a QWP, again used as optical isolator in order to discriminate between incident light and light that leaks out of the Fabry-Pérot interferometer. In addition, the laser beam is slightly focussed by a bi-convex lens (BCL3, focal length = 400 mm).

Next, the beam is mode matched coupled into the cavity<sup>8</sup> using a mode-matching optics built up by two plano-convex lenses<sup>9</sup> and a pin hole to discriminate against higher order transverse modes. Thereby, it is calculated with the freeware  $reZonator^{10}$  which lenses are to be used after analysing the laser beam incident on the cavity with a beam propagation analyser (*Ophir BeamSquared*) without any mode-matching optics in the path. Moreover, a third (bi-convex) lens (BCL4) is used if the calculation yields lens positions that are not possible to apply, i.e. because there is already something else placed.

In addition, one mirror is mounted on a piezo electric transducer (PET, PI S-316.10), which is driven by a controller (RampPET, PI E-727.3SD), that can apply ramp voltages and measures the current elongation of the piezo with sub-nm resolution.

<sup>&</sup>lt;sup>7</sup>Tiltable mirrors are indicated by TM and not tiltable mirrors are indicated by M, respectively.

<sup>&</sup>lt;sup>8</sup>The cavity is built up by two equal and highly reflective plano-concave mirrors with a radius of curvature of  $R_{\rm rad} = 1$  m and a mirror spacing of  $d_{\rm m} = 38.0$  cm. Depending on the wavelength of the tested laser, the mirrors are exchangeable. The following mirrors are used: (1) Layertec 140992 (R > 0.99995 @ 960 nm), (2) Layertec 140973 (R > 0.99995 @ 1045 nm) and (3) Layertec 140991 (R > 0.99995 @ 1260 nm). Thereby, the individual mirrors are suited to operate a few tens of nm around the specified wavelengths.

 $<sup>^{9}</sup>$ According to chapters (2.3) and (2.4) and equation (52), the focal lengths depend on the beam transformation necessary to match the transverse eigenmode of the cavity.

 $<sup>^{10}\</sup>mathrm{This}$  software uses the ABCD-matrix algorithm for Gaussian beams.

Furthermore, behind the cavity there is a fast photo diode (PD, *Femto HCA-S-200M-IN-FS*) with a bandwidth of 200 MHz to measure the transmitted light, that is focused on the PD by a lens (PCL2, focal length = 25 mm). The output voltage of the PD is observed by an oscilloscope (Scope, *Teledyne Lecroy WaveSurfer 10*) with a bandwidth of 1 GHz.

Moreover, a sawtooth-signal from a frequency generator (RampVCO, *KeySight 33510 B Waveform Generator*) can be fed into the VCO to control the frequency shift introduced to the laser light. Since the frequency of the VCO increases linearly with applied voltage, this has the effect of sweeping the frequency of the laser light (above a transmission fringe of the cavity in frequency domain). In addition, the from the VCO outgoing signal is observed on the Scope to measure its frequency and, therefore, the frequency shift introduced to the light.

In addition, to reduce the influence of mechanical vibrations, the whole setup is situated on an air-floated table and in order to reduce the influence of air turbulences, a beam path cover is used at the cavity, respectively.



Figure 25: Experimental setup of the cavity characterisation setup. The following symbols are used:  $\frac{\lambda}{2}$  (half-wave plate),  $\frac{\lambda}{4}$  (quarter-wave plate), A (aperture), AOM (acousto-optic modulator), AOM-DPC (AOM double-pass configuration), ASG (AOM stop-signal generator), BCL (bi-convex lens), BD (beam dump), M (mirror), MM (mode-matching optics), PBS (polarising beam splitter), PCL (plano-convex lens), PD (photo diode), PET (piezo electric transducer), RampPET (controller of the PET), RampVCO (waveform generator), Scope (oscilloscope), TM (tiltable mirror) and VCO (voltage controlled oscillator).

This experimental setup is used to measure the decay time  $\tau_c$  and the free spectral range  $\Delta \nu_{\rm FSR}$  of the used cavity. According to chapter (2.4), the decay time is measured by means of a cavity ring-down measurement.

To acquire cavity ring-down events, the laser beam frequency is increased by increasing the voltage applied at the VCO until a resonance frequency of the cavity is matched and high transmission is achieved. The piezo is not moved during this measurement. If the light, detected with the PD reaches a certain threshold  $V_{\text{threshold}}$ , the Scope is triggered, the laser beam is shut off and the exponentially decaying intensity is measured.

Since there would be ringing on the decaying intensity due to the beat between the rising cavity field and the incident laser beam whose frequency is still further shifted (out of resonance), it is necessary, that as soon as the field in the cavity builds up the incident beam is shut off. Here, it is not enough, that only a small fraction of light is coupled into the cavity in the off-resonance case and a shut down of the laser is needed [24].

Therefore, a self-built here called AOM stop-signal generator (ASG) is used, that observes the trigger-out port of the Scope. If the PD's output voltage reaches  $V_{\text{threshold}}$  and the Scope is triggered, a trigger pulse is applied at this port. As long as the ASG registers no trigger pulse, it generates an output voltage of approximately 1 V, that is fed to the modulation input port of the VCO, which drives the AOM. If at the VCO's modulation input port 1 V is applied, the VCO generates the necessary output signal to drive the AOM and laser light is diffracted into the first order to be used in the experiment (in AOM-DPC). If 0 V is applied at the VCO, the drive signal necessary for the AOM is shut down and, thus, the laser beam is shut off, since no light is diffracted into the first order any more and the zeroth order is blocked by an aperture (0 V: AOM off, 1 V: AOM on). Thereby, if the Scope is triggered, and the laser beam is shut off by means of applying 0 V at the VCO this low-level voltage is hold for a time longer than the cavity ring-down event. The ASG's standard output voltage is 1 V. The ASG is described in more detail below.

In order to determine the free spectral range  $\Delta \nu_{\rm FSR}$  of the cavity the AOM is driven by a fixed frequency (= constant applied voltage at the VCO's frequency port). In addition, in contrast to the measurement of the cavity ring-down curve, the mirror mounted on the PET is moved by means of the PET by at least a half wavelength to scan over more than one free spectral range. Simultaneously, the output voltage of the PD is measured with the Scope. This is repeated for another laser frequency by changing the voltage applied to the VCO's frequency port. In addition, in both cases the frequency of the signal fed into the AOM is measured with the Scope. By this, the laser is scanned by the Fabry-Pérot interferometer for two wavelengths with known frequency shift. Besides, the piezo elongation (or position) is measured by the piezo controller. From this, the free spectral range  $\Delta \nu_{\rm FSR}$  can be derived, as shown in chapter (3.2.1).

#### Now, the ASG is described in more detail.

It is used to shut down the laser if a certain voltage output level at the PD  $V_{\text{threshold}}$  is reached. Thereby, the laser is shut down by using the modulation input port of the AOM driver (the VCO) and switching the applied voltage from 1 V down to 0 V. Then, the laser beam is not diffracted into the first order any more and blocked by a beam blocker. Since this shut down has to be fast in comparison to the cavity decay time, fast electronics has to be used.

The ASG consists of the trigger out stage of the used oscilloscope, a voltage devider and a FPGA (*Xilinx Virtex-6* on a *Xilinx ML605* board). In figure (26) the circuit diagram is depicted and in figure (27) a photo of it is shown.

The trigger of the oscilloscope is set to  $V_{\text{threshold}}$ . As soon as the Scope is triggered, the trigger out stage generates an approximately 50 ns long square-wave signal with 3.3 V. Since this voltage level is to high for the used FPGA, it is reduced by a voltage devider by using resistors with 250  $\Omega$  and 750  $\Omega$ . Therefore, a suitable voltage level of 2.5 V results.

The signal is fed to the FPGA and converted by an input buffer (IBUF) into a logical signal. The signal leads to an OR-gate and to a finite state machine (Trigger FSM).

The combinatorial-only connection to the OR-gate results in a jitter-free processing of the trigger signal with a fixed delay. The output signal of the OR-gate is inverted by a NOT-gate and turned into an analog signal by the output buffer (OBUF). Thus, if the Scope is triggered the FPGA's output voltage, that is fed to the modulation input of the AOM driver, switches from 1.2 V down to 0 V and the laser is shut down<sup>11</sup>.

However, the Scope's trigger pulse only lasts about 50 ns (which is much shorter than the cavity ring-down

<sup>&</sup>lt;sup>11</sup>In figure (26) the OBUF generates a 2.5 V voltage level but the input impedance of the modulation port is only 50  $\Omega$ . Since the output stage of the FPGA is not able to hold the 2.5 V for a low impedance the signal voltage breaks down to 1.2 V that is close enough to the desired 1 V.

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event). Therefore, the Trigger FSM, which is driven by an oscillator at 66 MHz, samples the input signal in parallelly and generates its own output signal with the adjustable duration  $t_{\rm off}$ . When the Scope's trigger signal becomes low, that would lead to an output voltage back to 1.2 V, the FSM's output signal is already in a high state and the output voltage is hold at 0 V. Thus, the AOM remains shut off.

Combining both signals via the OR-gate results in a jitter-free rising edge signal with an adjustable duration. Thereby, the FSM's output signal duration  $t_{\text{off}}$  is set by a MicroBlaze SoftCore Processor, which is connected to the PC via an on-board USB-UART-Bridge. Therefore, the shut down signal's duration can be set in a convenient way via a terminal application on the PC.



Figure 26: Circuit diagram of the AOM stop-signal generator. This figure is described in the text above.

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Figure 27: Photo of the AOM stop-signal generator.

In figure (28) a measurement is shown, where the response time  $t_{ASG,resp}$  of the ASG was measured by triggering the Scope and feeding the output of the ASG back to the Scope. It yields  $t_{ASG,resp} = 47.9$  ns where the delay between triggering the Scope and the Scope's trigger out pulse already was 25 ns.



Figure 28: AOM stop-signal generator (ASG) electronic response time measurement.

In figure (29) the time between triggering the Scope and the laser intensity decay  $\tau_s$ , relevant for this experiment, was measured with a fast photo diode (*Thorlabs DET08CL/M*) with a bandwidth of 5 GHz. This time  $\tau_s$  was determined to be  $\tau_s = 1208$  ns.



Figure 29: AOM stop-signal generator (ASG) laser shut down time measurement. After the oscilloscope was triggered the laser intensity decayed down to 10% within 1208 ns.

### 3.1.2 Cavity length modulation measurement

This setup is used to determine the laser's linewidth  $\Delta \nu_0$  and is presented in figure (30).

The linearly polarised light of the tested laser is turned by a half-wave plate  $(\lambda/2)$  to be polarised horizontally. Next, the beam propagates through a beam splitter (BS), where a small fraction of the light is

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split off to be measured with a wavemeter ( $\lambda$ , *HighFinesse WS8-10*).

Simultaneously, the major part of the light passes a combination of polarising beam splitter (PBS) and quarter-wave plate ( $\lambda/4$ , QWP), which equals an optical isolator (independent of the propagation direction of the light with respect to the PBS, horizontally polarised light is transmitted and vertically polarised light is split off). Therefore, no light, that could influence the output spectrum of the laser, is reflected back.

Afterwards, the beam is coupled into the Fabry-Pérot interferometer<sup>12</sup> with mode-matching optics consisting of two plano-convex lenses<sup>13</sup> and a pinhole. In addition, a third lens (bi-convex lens BCL) is used to adjust the beam as well as possible to the cavity's fundamental mode. To find the best lenses regarding their focal lengths and lens positions, the same procedure as described in chapter (3.1.1) by means of usage of a beam propagation analyser and the freeware *reZonator* is used.

Moreover, one of the cavity mirrors is held by a piezo electric transducer (PET, *PI S-316.10*), which is driven by a controller (RampPET, *PI E-727.3SD*), that can apply ramp voltages to sweep the cavity mirror and, therefore, the cavity transmission fringe in frequency domain over the laser lineshape. Moreover, the controller is connected with a PC to detect the current elongation of the piezo from which the mirror velocity v can be derived.

The light transmitted through the Fabry-Pérot interferometer is focused by a bi-convex lens (BCL, focal length = 25.4 mm) on a photo diode (PD, *Femto HCA-S-200M-IN-FS*) with a bandwidth of 200 MHz. The PD is fed to an oscilloscope (Scope, *Teledyne Lecroy WaveSurfer 10*) connected with the PC to transfer the data to it. The Scope has a bandwidth of 1 GHz. Since the time of overlap of the cavity transmission fringe and the laser lineshape is of the order of a few  $\mu$ s, these high bandwidths are necessary.

In addition, to reduce the influence of mechanical vibrations, the whole setup is situated on an air-floated table and in order to reduce the influence of air turbulences, a beam path cover is used at the cavity, respectively.

 $<sup>^{12}</sup>$  The Fabry-Pérot interferometer is built up by two equal and highly reflective plano-concave mirrors with a radius of curvature of  $R_{\rm rad} = 1$  m and a mirror spacing of  $d_{\rm m} = 38.0$  cm. Depending on the wavelength of the tested laser, the mirrors are exchangeable. The following mirrors are used: (1) Layertec 140992 (R > 0.99995 @ 960 nm), (2) Layertec 140973 (R > 0.99995 @ 1045 nm) and (3) Layertec 140991 (R > 0.99995 @ 1260 nm). Thereby, the individual mirrors are suited to operate a few tens of nm around the specified wavelengths.

 $<sup>^{13}</sup>$ According to chapters (2.3) and (2.4) and equation (52), the focal lengths depend on the beam transformation necessary to match the eigenmode of the cavity.



Figure 30: Experimental setup of the cavity length modulation setup. The following symbols are used:  $\lambda$  (wavemeter),  $\frac{\lambda}{2}$  (half-wave plate),  $\frac{\lambda}{4}$  (quarter-wave plate), BCL (bi-convex lens), BD (beam dump), BS (beam splitter), MM (mode-matching optics), PBS (polarising beam splitter), PD (photo diode), PET (piezo electric transducer), RampPET (controller of the PET), Scope (oscilloscope) and TM (tiltable mirror).

The linewidth measurement is done by sweeping the cavity mirror mounted on the piezo with defined velocities v. Thereby, the intensity transmitted through the Fabry-Pérot interferometer is observed with respect to its maximum.

For convenience, during the linewidth measurement, the AOM-DPC used for the cavity characterisation setup is also integrated in the light path to build both setups as one. Thereby, during the whole measurement the AOM of the AOM-DPC is used with a constant frequency shift. Since the AOM-DPC is not used for this measurement, it is not depicted in figure (30).

### 3.1.3 The complete setup

As already mentioned, two measurements, the *cavity characterisation measurement* and the *cavity length modulation measurement* are necessary to measure the linewidth of the tested laser. Indeed, the two setups are built as one, but not every component is used in each measurement and in addition, the data analysis is completely separated. Therefore, for a better understanding and a clear presentation the two setups are treated separately further on.

Nevertheless, figure (31) shows the complete setup as built up in real life and in figure (32) a photo is depicted. Moreover, figure (33) shows a close-up photo without electronic devices, in figure (34) the ray
trace is depicted and figures (35) and (36) present the piezo and how it is connected to the beam path cover.



Figure 31: Complete setup as built in the lab. The following symbols are used:  $\lambda$  (wavemeter),  $\frac{\lambda}{2}$  (half-wave plate),  $\frac{\lambda}{4}$  (quarter-wave plate), A (aperture), AOM (acousto-optic modulator), AOM-DPC (AOM double-pass configuration), ASG (AOM stop-signal generator), BCL (bi-convex lens), BD (beam dump), M (mirror), MM (mode-matching optics), PBS (polarising beam splitter), PCL (plano-convex lens), PD (photo diode), PET (piezo electric transducer), RampPET (controller of the PET), RampVCO (waveform generator), Scope (oscilloscope), TM (tiltable mirror) and VCO (voltage controlled oscillator).



Figure 32: Photo of complete setup as built in the lab. 01: Fibre coupled and collimated output of tested laser, 02: Long-pass filter to filter out 532 nm pump light of the OPO-process, 03: HWP, 04: Lens, 05: PBS, 06: QWP, 07: Pinhole, 08: AOM, 09: Cavity input mirror, 10: Cavity output mirror mounted on the piezo, 11: Beam path cover, 12: PD, 13: Beam propagation analyser, 14: Scope, 15: Frequency generator, 16: Piezo driver, 17: ASG and 18: AOM Driver (VCO). Mirrors are not labelled.



Figure 33: Close-up photo of complete setup as built in the lab. 01: Fibre coupled and collimated output of tested laser, 02: Long-pass filter to filter out 532 nm pump light of the OPO-process, 03: HWP, 04: Lens, 05: PBS, 06: QWP, 07: Pinhole, 08: AOM, 09: Cavity input mirror, 10: Cavity output mirror mounted on the piezo, 11: Beam path cover and 12: PD. Mirrors are not labelled.

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Figure 34: Photo of complete setup as built in the lab with ray trace shown.



Figure 35: Photo of the cavity output mirror mounted on the piezo.



Figure 36: Installation of the cavity output mirror, mounted on the piezo, into the beam path cover.

## 3.2 Data acquisition and analysis

#### 3.2.1 Cavity characterisation measurement

Now, the results of the two cavity characterisation measurement are discussed. Thereby, the cavity decay time is determined with a typically used cavity ring-down measurement (CRDM) and the free spectral range (FSR) is obtained by a self-developed experiment.

Since the decay time is highly dependent on the mirror reflectivity depending on the wavelength the CRDM has to be done for each wavelength for that linewidths shall be measured. Here, it is only discussed for one wavelength because the measurement performance is independent from the wavelength. Moreover, the FSR measurement is also done for only one wavelength. For this experiment it is not necessary to perform it for all wavelengths since the differences in the optical round-trip path length, caused by the dielectric resonator mirrors coatings, is below the desired resolution.

First, the cavity decay time is determined.

According to chapter (2.4), the cavity decay time  $\tau_c$  is derived from the cavity ring-down measurement, where the intensity I(t) is measured after shutting down the light source.

In figure (37) such a measurement is shown that was measured for a laser wavelength of  $\lambda_0 = 1045$  nm. In addition, the function

$$y(t) = I_0 e^{-\frac{t}{\tau_c}} + I_N$$
(58)

is fitted to the data, where  $I_0$  is the maximal intensity and  $I_N$  is the intensity of noise and back ground light. The fit yields for the cavity decay time  $\tau_c = (2.329 \pm 0.015) \,\mu$ s, for the maximal intensity  $I_0 = 0.00940$  V and for the noise intensity  $I_N = 0.00290$  V with the statistical parameter  $R^2 = 0.98091$ . As uncertainty the standard deviation  $\sigma_{\tau c} = 0.015 \,\mu$ s is taken.



Figure 37: Cavity ring-down measurement to determine the cavity decay time with frequency scan done by the AOM. The exponential fit  $y(t) = I_0 \cdot \exp(-t\tau_c^{-1}) + I_N$  yields  $\tau_c = (2.329 \pm 0.015) \,\mu$ s with  $R^2 = 0.98091$ . The data was acquired without averaging.

Up to now only the statistical error  $\sigma_{\tau c} = 0.015 \,\mu s$  is included in the cavity decay time's uncertainty. From equations (32) and (33) an expression for the cavity decay time's uncertainty  $\delta \tilde{\tau}_c$  introduced by means of any change in the cavity length can be derived:

$$\delta \tilde{\tau}_{\rm c} = \left| \frac{\partial \tau_{\rm c}}{\partial d_{\rm m}} \cdot \delta d_{\rm m} \right| = \left| \frac{n_{\rm ref} F}{\pi c_0} \cdot \delta d_{\rm m} \right| \,. \tag{59}$$

Here, the optical round-trip path length  $L_{\text{opt}} = 2 n_{\text{ref}} d_{\text{m}}$  with the refractive index  $n_{\text{ref}}$  and the mirror spacing  $d_{\text{m}}$  and the finesse F were used.

The first uncertainty that has to be taken into account is the position stability of the used piezo. The observation of the position feedback given by the piezo controller, that, according to the manufacturer has an accuracy of 0.4 nm [34], yielded drifts of about  $\delta d_{\text{piezo}} = 50 \text{ nm}$  within 30 min.

Besides, the temperature and, therefore, the cavity mirror spacing are not fixed values<sup>14</sup>. Considering a change in temperature of  $\Delta T = 1$  K an additional uncertainty of the cavity mirror spacing due to thermal expansion  $\delta d_{\text{thermal}}$  can be calculated:

$$\delta d_{\text{thermal}} = \alpha_{\text{steel}} \cdot d_{\text{m}} \cdot \Delta T = 10.5 \cdot 10^{-6} \frac{1}{\text{K}} \cdot 38.0 \,\text{cm} \cdot 1 \,\text{K} = 3990 \,\text{nm} \,. \tag{60}$$

Thereby, the linear thermal expansion coefficient of steel  $\alpha_{\text{steel}} = 10.5 \cdot 10^{-6} \frac{1}{\text{K}}$  [35] and the mirror spacing  $d_{\text{m}} = 38.0 \text{ cm}$ , calculated below in equation (72), were used.

Now, the additional uncertainty of the cavity decay time reads:

$$\delta \tilde{\tau}_{\rm c} = \left| \frac{n_{\rm ref} F}{\pi c_0} \cdot (\delta d_{\rm piezo} + \delta d_{\rm thermal}) \right| = \left| \frac{1.00026 \cdot 7068}{\pi \cdot 2.998 \cdot 10^8 \, \frac{\rm m}{\rm s}} \cdot (50 \, \rm nm + 3990 \, \rm nm) \right| = 0.00003 \, \mu \rm s \,. \tag{61}$$

Here,  $n_{\rm ref} = 1.00026$  [36] and the finesse F = 7068, calculated below in equation (75), were used. The comparison of  $\delta \tilde{\tau}_{\rm c} = 0.00003 \,\mu {\rm s}$  and  $\sigma_{\tau \rm c} = 0.015 \,\mu {\rm s}$  shows that  $\delta \tilde{\tau}_{\rm c}$  is negligible in this context.

<sup>&</sup>lt;sup>14</sup>The cavity consits of two mirrors mounted on an optical table (*Newport*) made of steel.

Overall, ten cavity ring-down measurements were done, the decay times were calculated as described above and, then, the individual decay times were arithmetically averaged. This yields  $\bar{\tau}_{c} = (2.434 \pm 0.055) \,\mu$ s, where the bar indicates the averaged value, as for all following quantities.

In addition, a cavity ring-down measurement where the piezo was used to scan the frequency instead of using the  $AOM^{15}$  [23] was done.

The measurement result is shown in figure (38), where the acquired data was averaged 500 times. Again, the fit function given by equation (58) was fitted to the data. Due to the high averaging a decay time with much lower uncertainty results and is given by  $\tau_{c, \text{ piezo}} = (2.85562 \pm 0.00062) \,\mu\text{s}$ . Since the stated uncertainty  $(0.00062 \,\mu\text{s})$  is no longer large in comparison to  $\delta \tilde{\tau}_c = 0.00003 \,\mu\text{s}$  it is added to the uncertainty. Thus,  $\tau_{c, \text{ piezo}} = (2.85562 \pm 0.00065) \,\mu\text{s}$ .

Therefore, a discrepancy between the cavity decay times determined with AOM and piezo results, but it is not clear where it originates from. Probably it is caused by an insufficient mechanical stability of the built Fabry-Pérot interferometer that is discussed below.

Since this measurement serves the purpose to characterise the cavity for the linewidth measurement and the piezo is used there to scan the laser line shape instead of using the AOM, the value given by the piezo measurement is used in the following.



Figure 38: Cavity ring-down measurement to determine the cavity decay time with frequency scan done by the piezo. The exponential fit  $y(t) = I_0 \cdot \exp\left(-t\tau_c^{-1}\right) + I_N$  yields  $I_0 = 0.0139$  V,  $I_N = 0.00339$  V and  $\tau_c = (2.85562 \pm 0.00062) \,\mu$ s with  $R^2 = 0.99998$ . The data was acquired with 500 times averaging.

According to the condition of equation (26), namely that in case of cavity ring-down measurements the decay rate of the incident laser beam  $\gamma_s$  has to be large in comparison to the decay rate of the cavity  $\gamma_c$  ( $\gamma_s >> \gamma_c$ ), this condition seems to be fulfilled only in a weak way, because there is only a factor

 $<sup>^{15}</sup>$ Shifting the laser frequency with the AOM and holding the cavity resonance frequency constant is equal to holding the laser frequency constant and shifting the cavity resonance frequency with the piezo since both leads to the desired overlap between laser line shape and cavity transmission fringe in frequency domain.

of 2.4 between these values. This condition can be rewritten to  $\tau_c \gg \tau_s$ . With respect to figure (29) (measurement of the shut down time  $\tau_s$  of the laser incident on the cavity) it leads to:

$$\tau_{\rm c, \ piezo} = 2.85562 \,\mu {\rm s} \stackrel{?}{>} 1.208 \,\mu {\rm s} = \tau_{\rm s} \;.$$
 (62)

Nevertheless, on the decay curve taken with the piezo there is no ringing visible that would be the case if the laser is shut down to slowly [24].

Moreover, the free spectral range  $\Delta \nu_{\rm FSR}$  is calculated, whereby the following derivation is referred to figure (39).

monochromatic light If isin resonance with the Fabry-Pérot interferometer, the cavity length equals a multiple of the half wavelength of the light, because there are nodes of the electric field at the cavity end mirrors. Therefore, after increasing the cavity length by a half wavelength, the resonance condition is fulfilled again. Since in resonance the transmitted intensity is maximised, there will be periodically transmission of light if the length is raised. This is shown in figure (39) for two wavelengths  $\lambda_{0,0}$ (39 a) and  $\lambda_{0,1}$  (39 b). In addition, with  $x_{i,n}$  as the cavity length in case for which the  $n^{\text{th}}$  cavity resonance (for the  $i^{\text{th}}$  wavelength) is matching the laser wavelength, the following relation is given:

$$x_{i,n+1} = x_{i,n} + \frac{\lambda_{0,i}}{2}$$
. (63)

Moreover, by scanning both wavelengths of the laser and combining both scans in one plot, the situation shown in (39 c) will occur. Since in scanning mode the spectrum observed by the Fabry-Pérot interferometer is repeated periodically if the free spectral range is reached, scanning over more than one FSR will result in the presented picture. If the second wavelength  $\lambda_{0,1}$  has its origin in shifting the frequency of the first wavelength  $\lambda_{0,0}$  by  $\Delta\nu_{\rm ref}$  with an AOM, the following relationship is given, due to energy conservation:

$$\frac{c_0}{\lambda_{0,1}} = \frac{c_0}{\lambda_{0,0}} + \Delta\nu_{\text{ref}} . \qquad (64)$$



Figure 39: Determination of the Fabry-Pérot interferometer's free spectral range. The wavelengths  $\lambda_{0,0}$  and  $\lambda_{0,1}$  correspond to different frequency shifts introduced by the AOM. In addition,  $\Delta \nu_{\rm ref}$  is the frequency difference of  $\lambda_{0,0}$  and  $\lambda_{0,1}$ .

In a) the transmitted intensity against the piezo position for the wavelength  $\lambda_{0,0}$  is shown. Thereby,  $x_{0,n}$ ,  $x_{0,n+1}$  and  $x_{0,n+2}$ correspond to piezo positions for which the resonance condition is fulfilled. In b) the same plot but for the second wavelength  $\lambda_{0,1}$  with the resonance piezo positions  $x_{1,n}$ ,  $x_{1,n+1}$  and  $x_{1,n+2}$  is depicted. Moreover, in c) the signal curves of a) and b) are presented in one plot together.

If  $\Delta \nu_{\rm ref}$  is known and according to the fact that the FSR determines the width of the spectrum that can

be observed, an equation from which the FSR  $\Delta \nu_{\rm FSR}$  can be calculated, is given by:

$$\frac{x_{1,n} - x_{0,n}}{x_{0,n+1} - x_{0,n}} = \frac{\Delta\nu_{\text{ref}}}{\Delta\nu_{\text{FSR}}} .$$
(65)

Here, the assumption that  $\Delta \nu_{\text{FSR}}$  does nearly not change with the increase of the cavity length, was made. Indeed,  $\Delta \nu_{\text{FSR}}$  changes only by 0.00019% if the length is increased by 650 nm for a cavity length of  $d_{\text{m}} = 34 \text{ cm}$ . Therefore,  $\Delta \nu_{\text{FSR}}$  is given by:

$$\Delta \nu_{\rm FSR} = \frac{x_{0,n+1} - x_{0,n}}{x_{1,n} - x_{0,n}} \Delta \nu_{\rm ref} .$$
(66)

In addition, it should be outlined, that in this derivation and in the theory chapter, all formulas were derived with a refractive index  $n_{ref} = 1$ . Nevertheless, the real refractive index, although unknown, is included as an increased cavity length in all measured quantities, or it is cancelled out because of the fraction as in equation (65). Any refractive index used in the calculations serves the purpose to estimate the uncertainties of measured quantities, only. The optimum values are not affected.

The measurement was done with a laser wavelength of  $\lambda_0 = 1090 \,\mathrm{nm}$ . The measurement of the piezo positions, for which a maximum in transmission is observed, yields  $x_{0,n+1} = (610.8 \pm 0.4) \,\mathrm{nm}$ ,  $x_{0,n} = (66.1 \pm 0.4) \,\mathrm{nm}$  and  $x_{1,n} = (115.0 \pm 0.4) \,\mathrm{nm}$ . Since the laser is always twice frequency shifted if incident on the cavity (AOM-DPC) and the difference between the two wavelengths used for the measurement results from driving the AOM with two different frequencies  $\nu_{AOM,1}$  and  $\nu_{AOM,2}$ , respectively, the reference frequency  $\Delta \nu_{\rm ref}$  equals two times the difference between the AOM frequencies, which were measured by feeding the signal of the VCO to the AOM and to the oscilloscope. Thus, the reference frequency  $\Delta \nu_{\rm ref}$  is given by:

$$\Delta \nu_{\rm ref} = 2\nu_{\rm AOM,2} - 2\nu_{\rm AOM,1} = 2 \cdot (\nu_{\rm AOM,2} - \nu_{\rm AOM,1}) = 2 \cdot (150.6709 \,\,\mathrm{MHz} - 133.1231 \,\,\mathrm{MHz}) = 35.0956 \,\,\mathrm{MHz} \,, \tag{67}$$

with  $\nu_{AOM,1} = 133.1231 \text{ MHz}$  and  $\nu_{AOM,2} = 150.6709 \text{ MHz}$ . Moreover, the uncertainty of the reference frequency  $\delta \Delta \nu_{ref}$  is calculated by:

$$\delta\Delta\nu_{\rm ref} = \left|\frac{\partial\Delta\nu_{\rm ref}}{\partial\nu_{\rm AOM,1}} \cdot \delta\nu_{\rm AOM,1}\right| + \left|\frac{\partial\Delta\nu_{\rm ref}}{\partial\nu_{\rm AOM,2}} \cdot \delta\nu_{\rm AOM,2}\right| = \left|-2 \cdot \delta\nu_{\rm AOM,1}\right| + \left|2 \cdot \delta\nu_{\rm AOM,2}\right| \\ = 2 \cdot \left(\delta\nu_{\rm AOM,1} + \delta\nu_{\rm AOM,2}\right) = 2 \cdot \left(62.2\,\rm{kHz} + 60.3\,\rm{kHz}\right) = 245.0\,\rm{kHz}$$
(68)

with the uncertainties of the AOM frequencies  $\delta \nu_{AOM,1} = 62.2 \text{ kHz}$  and  $\delta \nu_{AOM,2} = 60.3 \text{ kHz}$  measured with the oscilloscope. Thus,  $\Delta \nu_{ref} = (35.0956 \pm 0.2450) \text{ MHz}$ . Now,  $\Delta \nu_{FSR}$  can be calculated.

$$\Delta\nu_{\rm FSR} = \frac{x_{0,n+1} - x_{0,n}}{x_{1,n} - x_{0,n}} \Delta\nu_{\rm ref} = \frac{610.8\,\rm{nm} - 66.1\,\rm{nm}}{115.0\,\rm{nm} - 66.1\,\rm{nm}} \cdot 35.0956\,\rm{MHz} = 390.9\,\rm{MHz} \tag{69}$$

In addition, the uncertainty of the free spectral range  $\delta \Delta \hat{\nu}_{FSR}$  following from this calculation is given by:

$$\begin{split} \delta\Delta\hat{\nu}_{\text{FSR}} &= \left| \frac{\partial\Delta\nu_{\text{FSR}}}{\partial x_{0,n+1}} \cdot \delta x_{0,n+1} \right| + \left| \frac{\partial\Delta\nu_{\text{FSR}}}{\partial x_{0,n}} \cdot \delta x_{0,n} \right| + \left| \frac{\partial\Delta\nu_{\text{FSR}}}{\partial x_{1,n}} \cdot \delta x_{1,n} \right| + \left| \frac{\partial\Delta\nu_{\text{FSR}}}{\partial\Delta\nu_{\text{ref}}} \cdot \delta\Delta\nu_{\text{ref}} \right| \\ &= \left| \frac{1}{x_{1,n} - x_{0,n}} \Delta\nu_{\text{ref}} \cdot \delta x_{0,n+1} \right| + \left| \frac{x_{0,n+1} - x_{1,n}}{(x_{1,n} - x_{0,n})^2} \Delta\nu_{\text{ref}} \cdot \delta x_{0,n} \right| \\ &+ \left| -\frac{x_{0,n+1} - x_{0,n}}{(x_{1,n} - x_{0,n})^2} \Delta\nu_{\text{ref}} \cdot \delta x_{1,n} \right| + \left| \frac{x_{0,n+1} - x_{0,n}}{x_{1,n} - x_{0,n}} \cdot \delta\Delta\nu_{\text{ref}} \right| \\ &= \left| \frac{1}{115.0 \text{ nm} - 66.1 \text{ nm}} \cdot 35.0956 \text{ MHz} \cdot 0.4 \text{ nm} \right| + \left| \frac{610.8 \text{ nm} - 115.0 \text{ nm}}{(115.0 \text{ nm} - 66.1 \text{ nm})^2} \cdot 35.0956 \text{ MHz} \cdot 0.4 \text{ nm} \right| \\ &+ \left| -\frac{610.8 \text{ nm} - 66.1 \text{ nm}}{(115.0 \text{ nm} - 66.1 \text{ nm})^2} \cdot 35.0956 \text{ MHz} \cdot 0.4 \text{ nm} \right| + \left| \frac{610.8 \text{ nm} - 66.1 \text{ nm}}{115.0 \text{ nm} - 66.1 \text{ nm}} \cdot 0.2450 \text{ MHz} \right| \\ &= 9.1 \text{ MHz} \,. \end{split}$$

It follows  $\Delta \nu_{\text{FSR}} = (390.9 \pm 9.1) \text{ MHz}.$ 

Since the cavity free spectral range  $\Delta \nu_{\rm FSR} = c_0 (2 n_{\rm ref} d_{\rm m})^{-1}$  depends on the cavity length, the uncertainty of the mirror spacing  $\delta d_{\rm piezo} + \delta d_{\rm thermal} = 4040 \,\mathrm{nm}$  has to be taken into account as in case of the cavity decay time. This error  $\delta \Delta \tilde{\nu}_{\rm FSR}$  follows from:

$$\delta\Delta\tilde{\nu}_{\text{FSR}} = \left| \frac{\partial\Delta\nu_{\text{FSR}}}{\partial d_{\text{m}}} \cdot \delta d_{\text{m}} \right| = \left| -\frac{c_0}{2 \, n_{\text{ref}} \, d_{\text{m}}^2} \cdot \left( \delta d_{\text{piezo}} + \delta d_{\text{thermal}} \right) \right|$$
$$= \left| -\frac{2.998 \cdot 10^8 \, \frac{\text{m}}{\text{s}}}{2 \cdot 1.00026 \cdot (38.0 \, \text{cm})^2} \cdot 4040 \, \text{nm} \right|$$
$$= 0.00009 \, \text{MHz} \,, \tag{71}$$

where the refractive index  $n_{\rm ref} = 1.00026$  [36] and the mirror spacing  $d_{\rm m} = 38.0$  cm, calculated below in equation (72), were used. Since this additional uncertainty due to thermal and piezo drifts is small with respect to the result of equation (70) it is neglected.

Again,  $\Delta \nu_{\text{FSR}}$  was measured ten times and averaged arithmetically, which leads to  $\overline{\Delta \nu}_{\text{FSR}} = (393.9 \pm 9.6) \text{ MHz}.$ 

In addition, for the calculation of the uncertainties of cavity decay time and free spectral range, the mirror spacing  $d_{\rm m}$  is needed and can be derived from  $\Delta \nu_{\rm FSR} = c_0 L_{\rm opt}^{-1} = c_0 (2 n_{\rm ref} d_{\rm m})^{-1}$ . With refractive index  $n_{\rm ref} = 1.00026$  [36] the mirror spacing  $d_{\rm m}$  follows:

$$d_{\rm m} = \frac{c_0}{2 n_{\rm ref} \overline{\Delta \nu}_{\rm FSR}} = \frac{2.998 \cdot 10^8 \, \frac{\rm m}{\rm s}}{2 \cdot 1.00026 \cdot 393.9 \, \rm MHz} = 38.0 \,\rm cm \;. \tag{72}$$

Besides, as a control result for the FSR  $\overline{\Delta\nu}_{\text{FSR}}$ , the FSR is determined by using the distance  $d_{\text{m}}$  between the mirrors measured with a ruler and calculating the FSR with equation (33). Using the refractive index of air  $n_{\text{ref}} = 1.00026$  [36], the vacuum speed of light  $c_0 = 2.998 \cdot 10^8 \frac{m}{s}$  and the measured distance between the mirrors  $d_{\text{m}} = (38.0 \pm 0.1)$  cm, this yields:

$$\Delta\nu_{\rm FSR, \ ruler} = \frac{c_0}{2 \cdot n_{\rm ref} \cdot d_{\rm m}} = \frac{2.998 \cdot 10^8 \frac{m}{s}}{2 \cdot 1.00026 \cdot 38.0 \,\rm cm} = 394.3 \,\rm MHz \ , \tag{73}$$

with the uncertainty  $\delta \Delta \nu_{\rm FSR}$  given by:

$$\delta\Delta\nu_{\rm FSR, \ ruler} = \left|\frac{\partial\Delta\nu_{\rm FSR}}{\partial d_{\rm m}} \cdot \delta d_{\rm m}\right| = \left|-\frac{c_0}{2 \cdot n_{\rm ref} \cdot d_{\rm m}^2} \cdot \delta d_{\rm m}\right| = \left|-\frac{2.998 \cdot 10^8 \,\frac{\rm m}{\rm s}}{2 \cdot 1.00026 \cdot (38.0 \,\mathrm{cm})^2} \cdot 0.1 \,\mathrm{cm}\right| = 1.0 \,\mathrm{MHz} \,, \tag{74}$$

where the uncertainty  $\delta d_{\rm m} = 0.1 \,\mathrm{cm}$  is used. Thus, the FSR  $\Delta \nu_{\rm FSR, \ ruler} = (394.3 \pm 1.0) \,\mathrm{MHz}$  measured with a ruler and the FSR  $\overline{\Delta \nu}_{\rm FSR} = (393.9 \pm 9.6) \,\mathrm{MHz}$  determined prior are in compliance within their uncertainties.

Now, according to equation (32), the finesse F of the Fabry-Pérot interferometer can be calculated by using the cavity decay time  $\tau_{\rm c, \ piezo} = (2.85562 \pm 0.00065) \,\mu \text{s}$  and the free spectral range  $\overline{\Delta\nu}_{\rm FSR} = (393.9 \pm 9.6) \,\text{MHz}$ :

$$F = 2\pi \overline{\Delta\nu}_{\rm FSR} \tau_{\rm c, \ piezo} = 2\pi \cdot 393.9 \,\text{MHz} \cdot 2.85562 \,\mu\text{s} = 7068 \;. \tag{75}$$

Besides, the uncertainty of the finesse  $\delta F$  is derived from:

$$\delta F = \left| \frac{\partial F}{\partial \overline{\Delta \nu}_{\text{FSR}}} \cdot \delta \overline{\Delta \nu}_{\text{FSR}} \right| + \left| \frac{\partial F}{\partial \tau_{\text{c, piezo}}} \cdot \delta \tau_{\text{c, piezo}} \right|$$
$$= \left| 2\pi \tau_{\text{c, piezo}} \cdot \delta \overline{\Delta \nu}_{\text{FSR}} \right| + \left| 2\pi \overline{\Delta \nu}_{\text{FSR}} \cdot \delta \tau_{\text{c, piezo}} \right|$$
$$= \left| 2\pi \cdot 2.85562 \,\mu\text{s} \cdot 9.6 \,\text{MHz} \right| + \left| 2\pi \cdot 393.9 \,\text{MHz} \cdot 0.00065 \,\mu\text{s} \right|$$
$$= 174 \,. \tag{76}$$

Thus, the finesse is given by  $F = 7068 \pm 174$  and the Fabry-Pérot interferometer is fully characterised.

#### 3.2.2 Cavity length modulation measurement

From this measurement, the linewidth of the tested laser  $\Delta \nu_0$  is determined.

According to equation (41), the magnitude of the fringe MF is recorded for the varied mirror velocity v. Thereby, the mirror intensity transmission T, the mirror intensity reflectance R, the cavity decay time  $\tau_c$ , the cavity round-trip time  $t_r$  and the laser frequency  $\nu_0$  are necessary quantities for the calculation of the theoretical curves which are fitted to the experimentally measured data.

The cavity decay time  $\tau_{\rm c, \ piezo} = (2.85562 \pm 0.00065) \,\mu s$  is already measured.

In addition, the cavity round trip time  $t_r$  is given by [9]:

$$t_{\rm r} = \frac{1}{\overline{\Delta\nu}_{\rm FSR}} \ . \tag{77}$$

With  $\overline{\Delta\nu}_{\text{FSR}} = (393.9 \pm 9.6) \text{ MHz}$  follows  $t_{\text{r}} = (2.539 \pm 0.062) \text{ ns}$  where the uncertainty  $\delta t_{\text{r}}$  was calculated by:

$$\delta t_{\rm r} = \left| \frac{\partial t_{\rm r}}{\partial \overline{\Delta} \nu_{\rm FSR}} \cdot \delta \overline{\Delta} \overline{\nu}_{\rm FSR} \right| = \left| -\frac{1}{\overline{\Delta} \overline{\nu}_{\rm FSR}^2} \cdot \delta \overline{\Delta} \overline{\nu}_{\rm FSR} \right| = \left| -\frac{1}{(393.9 \,\mathrm{MHz})^2} \cdot 9.6 \,\mathrm{MHz} \right| = 0.062 \,\mathrm{ns} \,. \tag{78}$$

Moreover, the laser wavelength  $\lambda_0 = (1044.802415 \pm 0.000036)$  nm was measured with a wavemeter<sup>16</sup>. Since the refractive index is already included in the measured value of the wavelength, the laser frequency  $\nu_0$  is given by:

$$\nu_0 = \frac{c_0}{\lambda_0} = \frac{2.998 \cdot 10^8 \, \frac{m}{s}}{1044.802415 \, \text{nm}} = 286.93699 \, \text{THz} \,. \tag{79}$$

The uncertainty of the laser frequency  $\delta \nu_0$  is stated as follows:

$$\delta\nu_{0} = \left|\frac{\partial\nu_{0}}{\partial\lambda_{0}} \cdot \delta\lambda_{0}\right| = \left|-\frac{c_{0}}{\lambda_{0}^{2}} \cdot \delta\lambda_{0}\right| = \left|-\frac{2.998 \cdot 10^{8} \frac{m}{s}}{(1044.802415 \,\mathrm{nm})^{2}} \cdot 0.000036 \,\mathrm{nm}\right| = 0.00001 \,\mathrm{THz} \,. \tag{80}$$

where  $\delta \lambda_0$  is the uncertainty of the wavelength. Therefore, the laser frequency is determined to  $\nu_0 = (286.93699 \pm 0.00001)$  THz.

As already mentioned, the mirror intensity reflection coefficient R is needed. It can be derived by solving equation (31) numerically:

$$F = \frac{\pi\sqrt{R}}{1-R} \ . \tag{81}$$

Therefore, equation (81) is solved for a set of mirror reflectivities  $R_k$  with  $R_k \in \{R_k = k \cdot 10^{-7} \text{ with } k \in \{0, 1, 2, ..., 10^7 - 1\}\}$ :

$$F_{\mathbf{k}} = \frac{\pi \sqrt{R_{\mathbf{k}}}}{1 - R_{\mathbf{k}}} \,. \tag{82}$$

As mirror reflectivity the  $R_k$  value is chosen, that leads to the smallest deviation between  $F_k(R_k)$  and F = 7068, where the deviation is much smaller than the uncertainty of F. In addition, the uncertainty of the mirror reflectivity is calculated by repeating this procedure for  $F \to F - \delta F = 7068 - 174 = 6894$  and  $F \to F + \delta F = 7068 + 174 = 7242$ , respectively. It follows  $R = 0.999556 \pm 0.000011$ .

Furthermore, the mirror intensity transmission T ensues from T = 1 - R as  $T = 0.000444 \pm 0.000011$ .

Thereby, the determined mirror reflectivity of  $R = 0.999556 \pm 0.000011$  is not in agreement with the reflectivity R = 0.99995 stated by the manufacturer. The reason for this is a problem with the mechanical stability of the Fabry-Pérot interferometer, that is discussed below.

 $<sup>^{16}{\</sup>rm The}$  used wavemeter HighFinesse~WS8-10 has a specified uncertainty of 10 MHz, which equals  $0.000036\,{\rm nm}$  at a wavelength of  $1045\,{\rm nm}.$ 

With these values of  $\tau_{c, \text{ piezo}}$ ,  $t_r$ ,  $\nu_0$ , R and T equation (41) is solved numerically for a set of potential laser linewidths  $\Delta \nu_0$  with a step size of  $\delta \Delta \nu_0 = 10 \text{ kHz}$  against the mirror velocity v. Thereby, a detailed description of the numerical calculation is shown in section (5.1). Moreover, for each theoretically calculated  $MF_{\Delta\nu_0}(v)$  curve the  $\chi^2 [MF_{\Delta\nu_0}(v)]$  value given by equation (83) is calculated. From that  $MF_{\Delta\nu_0}(v)$ curve the lowest value of:

$$\chi^{2} \left[ MF_{\Delta\nu_{0}}(v) \right] = \sum_{\mathbf{k}} \frac{\left[ \alpha f_{\mathbf{e}}(v_{\mathbf{k}}) - f_{\mathbf{c}}(v_{\mathbf{k}}) \right]^{2}}{f_{\mathbf{c}}(v_{\mathbf{k}})}$$
(83)

is obtained, the linewidth  $\Delta \nu_0$  is chosen as linewidth of the tested laser, since this  $MF_{\Delta\nu_0}(v)$  curve describes the measured data best. Thereby,  $\alpha$  is a scale factor for the calculation to adjust the measured intensity to the calculated one. Moreover,  $f_{\rm e}(v_{\rm k})$  is the experimentally measured magnitude of the fringe for the k<sup>th</sup> velocity  $v_{\rm k}$  and  $f_{\rm c}(v_{\rm k}) = MF_{\Delta\nu_0}(v_{\rm k})$  the theoretically calculated value for the k<sup>th</sup> velocity  $v_{\rm k}$ , respectively [6].

Due to stability problems with the built Fabry-Pérot interferometer (FPI), that are discussed below, no successful measurement could be done to determine the linewidth with the cavity length modulation technique. Therefore, to illustrate the determination of the linewidth using equation (83), a labview routine was programmed in order to generate theoretically calculated  $MF_{\Delta\nu_0}(v)$  curves for potential linewidths  $\Delta\nu_0$  of 210 kHz, 230 kHz, 250 kHz, 270 kHz and 290 kHz using the experimentally measured parameters for  $\tau_{\rm c, \ piezo}$ ,  $t_{\rm r}$ ,  $\nu_0$ , R and T. Then, a second labview routine was programmed to fit these curves to the 250 kHz linewidth curve that was interpreted as measured data leading to  $\chi^2$  values (depending on the linewidth) as shown in figure (40). I.e. for the 250 kHz linewidth  $MF_{\Delta\nu_0=210 \text{ kHz}}(v)$  curve the  $\chi^2 [MF_{\Delta\nu_0=210 \text{ kHz}}(v)]$  was calculated. Then, for the 250 kHz linewidth MF(v) curve interpreted as measured data and the theoretically calculated 210 kHz linewidth MF(v) curve interpreted as measured data and the theoretically calculated 210 kHz linewidth MF(v) curve interpreted as measured data and the theoretically calculated 230 kHz linewidth  $MF_{\Delta\nu_0=230 \text{ kHz}}(v)$  curve the  $\chi^2 [MF_{\Delta\nu_0=230 \text{ kHz}}(v)]$  was calculated and so on.



Fit of theoretically calculated to simulated measurement data

Figure 40: Calculated  $\chi^2$  values [given by equation (83)] for theoretically calculated  $MF_{\Delta\nu_0}(v)$  curves for potential linewidths  $\Delta\nu_0$  equal to 210 kHz, 230 kHz, 250 kHz, 270 kHz and 290 kHz and the theoretically calculated 250 kHz linewidth MF(v) curve which was interpreted as measured data. Since the lowest  $\chi^2$ value occurs for the linewidth 250 kHz (as expected) this linewidth would be chosen as linewidth of the tested laser.

As soon as the stability problems are solved, the resolution of the linewidth determination using the cavity length modulation technique (CLMT) is of great importance. Therefore, the expectable resolution is discussed, now.

According to equation (32) ( $\tau_{\rm c} \cdot \Delta \nu_{\rm FSR} = \text{const.}$  and  $\tau_{\rm c} \propto d_{\rm m}$ ), in case of an increase of the mirror spacing  $d_{\rm m}$  the cavity decay time (DT)  $\tau_{\rm c}$  is increased and the free spectral range (FSR)  $\Delta \nu_{\rm FSR}$  is decreased, and vice versa in case of a decreased mirror spacing  $d_{\rm m}$ .

Thus, to consider the effect of the uncertainties of measured DT  $\tau_{c, piezo} = \tau_{c,0} \pm \delta \tau_c = (2.85562 \pm 10^{-5})$ 0.00065)  $\mu$ s and FSR  $\overline{\Delta\nu}_{FSR} = \Delta\nu_{FSR,0} \pm \delta\Delta\nu_{FSR} = (393.9 \pm 9.6)$  MHz on the uncertainty of the linewidth determined by the CLMT, it is possible to calculate the  $MF_{\Delta\nu_0}(v)$  curves for the potential linewidths  $\Delta \nu_0 = 210 \text{ kHz}, 230 \text{ kHz}, 250 \text{ kHz}, 270 \text{ kHz}$  and 290 kHz using the optimum values of measured DT ( $\tau_{c,0} = 2.85562 \,\mu s$ ) and FSR ( $\Delta \nu_{FSR,0} = 393.9 \,\text{MHz}$ ) and compare this with calculated  $MF_{\Delta\nu_0}(v)$  curves using the minimum [maximum] DT ( $\tau_{c,0} - \delta\tau_c = 2.85562 \,\mu s - 0.00065 \,\mu s =$  $2.85497 \,\mu s$  [ $(\tau_{c,0} + \delta \tau_c = 2.85562 \,\mu s + 0.00065 \,\mu s = 2.85630 \,\mu s$ ] and the maximum [minimum] FSR  $(\Delta\nu_{\rm FSR.0} + \delta\Delta\nu_{\rm FSR} = 393.9\,\rm{MHz} + 9.6\,\rm{MHz} = 403.5\,\rm{MHz}) \left[ (\Delta\nu_{\rm FSR,0} - \delta\Delta\nu_{\rm FSR} = 393.9\,\rm{MHz} - 9.6\,\rm{MHz} = 403.5\,\rm{MHz} \right]$ 384.3 MHz)].

Then, from a graphical presentation it can be estimated, which  $MF_{\Delta\nu_0}(v)$  curves are still distinguishable and which are  $not^{17}$ .

This is depicted in figure (41) and yields an uncertainty of 1 kHz if only the uncertainties of the quantities describing the FPI (DT, FSR) are considered. Not considered are any uncertainties of the transmitted intensity measured with the photo diode.



## Consideration of the uncertainty of the CLMT

Figure 41: Consideration of the uncertainty of the cavity length modulation measurement. The stated magnitude of the fringe corresponds to the maximum fraction of incident light that is transmitted for any time. The  $MF_{\text{center}}(250 \text{ kHz})$  curve is nearly overlapped with the  $MF_{\text{IncLen}}(250 \text{ kHz})$  and  $MF_{\rm DecLen}(250\,\rm kHz)$  curves.

It ensues that the uncertainties of cavity decay time and free spectral range can be neglected. Hence, as

 $<sup>^{17}</sup>$ Here it was also tested if the deviation in case of an increased DT and an increased FSR leads to a higher deviation from the best value curve but the deviation was even smaller.

uncertainty of the linewidth the fundamental limit given by the equations (36) and (37), respectively, is used. Besides, the relation  $\overline{\Delta\nu}_{\rm FSR} = c_0 L_{\rm opt}^{-1}$  is utilised. Thus:

$$R_{\rm FPI} = \frac{\tilde{\nu}_0}{\Delta \tilde{\nu}_0} = \frac{c_0 F}{\lambda_0 \overline{\Delta \nu}_{\rm FSR}} = \frac{\nu_0 F}{\overline{\Delta \nu}_{\rm FSR}} .$$
(84)

 $\tilde{\nu}_0$  and  $\Delta \tilde{\nu}_0$  are the considered frequency and frequency deviation, respectively,  $\lambda_0$  is the laser wavelength,  $c_0$  the speed of light and  $L_{\text{opt}}$  the optical round-trip path length.

Therefore, the minimal resolvable deviation in frequency referring to the laser frequency  $\nu_0$  with  $\tilde{\nu}_0 = \nu_0$  reads:

$$\Delta \tilde{\nu}_0 = \frac{\overline{\Delta \nu}_{\rm FSR}}{F} . \tag{85}$$

Now, for the uncertainty of the laser linewidth  $\delta\Delta\nu_0$  the uncertainty  $\delta\Delta\tilde{\nu}_0$  of  $\Delta\tilde{\nu}_0$  is taken into account, too. This leads to:

$$\begin{split} \delta\Delta\nu_{0} &= \Delta\tilde{\nu}_{0} + \delta\Delta\tilde{\nu}_{0} \\ &= \Delta\tilde{\nu}_{0} + \left| \frac{\partial\Delta\tilde{\nu}_{0}}{\partial\overline{\Delta}\nu_{\rm FSR}} \cdot \delta\overline{\Delta}\overline{\nu}_{\rm FSR} \right| + \left| \frac{\partial\Delta\tilde{\nu}_{0}}{\partial F} \cdot \delta F \right| \\ &= \frac{\overline{\Delta}\nu_{\rm FSR}}{F} + \left| \frac{1}{F} \cdot \delta\overline{\Delta}\overline{\nu}_{\rm FSR} \right| + \left| -\frac{\overline{\Delta}\nu_{\rm FSR}}{F^{2}} \cdot \delta F \right| \\ &= \frac{393.9 \,\mathrm{MHz}}{7068} + \frac{9.6 \,\mathrm{MHz}}{7068} + \frac{393.9 \,\mathrm{MHz} \cdot 174}{(7068)^{2}} \end{split}$$
(86)  
$$&= 58 \,\mathrm{kHz} \;. \end{split}$$

As a result the resolution of the CLMT would be  $58\,\mathrm{kHz}$  if the fluctuations of the cavity transmission fringes were reduced far enough.

Within this thesis it was not possible to measure the linewidth with the built Fabry-Pérot interferometer (FPI) due to very large fluctuations of the heights of the transmission peaks. Thereby, the maximum height of a transmission peak equals the MF-value, and even if averaging is used it has to be stable within a certain range to achieve the desired resolution, since the differences between the MF-values for linewidths that are only a few kHz separated are small [see figure (41)].

Now, the origin of those fluctuations is investigated.

First, with the FPI a complete FSR was scanned for a laser wavelength of  $\lambda_0 = 1045 \text{ nm}$  (using *C-WAVE* #023) and the measured signal curve is shown in figure (42).



Figure 42: Transmission signal curve for scanning over more than one FSR. It was taken using *C*-WAVE at a wavelength of  $\lambda_0 = 1045$  nm with the setup in the path [the standard measurement condition as represented in figure (31)] with a scan speed of v = 12 FSR/s. The two high peaks correspond to the (q,0,0) and (q+1,0,0) FPI modes.

Although more than one FSR is shown with two resonances with approximately the same peak height, this is by far not reproducible with each scan. Thereby, the resonance heights vary extremely. If considering one of the two shown resonances, starting with the shown resonance height as 100%, for the next scans possibly 10%, 50%, 80%, 20%, 110%, 20%, 25%, 40%, 10% and 70% is observable, whereby no pattern is discernible and the peak heights seem to vary randomly. Only one of a few tens of taken signal curves shows the depicted situation and the variations are far above those expected from the signal-to-noise ratio.

Therefore, the individual resonances are investigated further on, regarding the origin of those fluctuations. In the following, the shown signal curves always correspond to **one** of the main resonances shown in figure (42). Thereby, different measurement conditions were applied. The laser beam was coupled into the FPI with and without the complete setup in the light path to check influences of the setup, the scan velocities were varied to examine any possible time dependence and a second laser, but not another *C*-WAVE was used, to validate a potential problem with the laser source.

First, a theoretically calculated signal curve given for the measurement parameters [using equation (41)] is depicted in figure (43).



Figure 43: Theoretical signal curve of transmitted intensity. Calculated for the parameters: Scan velocity v = 15 FSR/s, wavelength  $\lambda_0 = 1045 \text{ nm}$ , free spectral range  $\overline{\Delta}\nu_{\text{FSR}} = 393.9 \text{ MHz}$  and laser linewidth  $\Delta\nu_0 = 200 \text{ kHz}$ . The curve shows a small amount of asymmetry that increases with increasing scan speed (not shown here) but the general signal curve shape stays the same.

Now, measurement data is shown.

Thereby, the frequency scanning was done with the piezo driven by a ramp-voltage (as in all cases where the piezo was used to scan the frequency) and the oscilloscope was triggered each time the piezo reached a certain position (using the position feedback of the piezo controller with a resolution of 0.4 nm and the piezo controllers trigger out function) where only data was taken during the rising edge of the ramp voltage.

In each case for that resonances are shown, they were chosen from at least 20 observed resonances. Thereby, resonances representing the ensemble of measured resonances, because they show characteristics (number of peaks, ...) that are typical for the individual measurement parameters, are depicted.

In figure (44) the comparison of the observed transmission fringes in cases of the whole setup (AOM-DPC, ...) is in the path of the laser beam incident on the Fabry-Pérot interferometer (right side) and the beam is coupled into the FPI only with mode-matching optics in the path (left side), respectively, is shown. It was measured with C-WAVE #023 at 1045 nm and a piezo velocity of 15 FSR/s. Thereby, the question is if there are differences between the resonances for the two cases that would lead to the interpretation that something in the setup causes the deviation from the theoretically expected signal curve [shown in figure (43)].



Figure 44: Comparison of the observed transmission fringes in cases of the whole setup (AOM-DPC path,  $\dots$ ) is in the path of the laser beam (right) and the laser beam is coupled into the FPI only with modematching optics in the path (left). Thereby, always one main resonance [i.e. the (q,0,0) mode] is shown. In both cases the main resonances decompose in many sub peaks.

Since, the main resonances decompose in many sub peaks and vary in their number of peaks and peak heights independent of the cases of the whole setup (AOM-DPC, ...) is in the path and the laser beam is incident directly on the FPI, an influence of the setup is very unlikely.

In order to justify the exclusion of the setup as reason for the observed fluctuations of the transmission fringe heights further, the beam pointing stability and laser's power stability directly in front of the FPI were measured.

The short time power stability in front of the cavity was measured with a *Thorlabs DET08CL/M* photodiode (5 GHz bandwidth) and the *Teledyne Lecroy WaveSurfer 10* oscilloscope (1 GHz bandwidth) using *C-WAVE #023* at a wavelength of  $\lambda_0 = 1045$  nm. The result is shown in figure (45).



Figure 45: Power stability in front of the cavity. The standard deviation is only  $0.09\,\%$  of the averaged power.

The standard deviation is only 0.09% of the averaged power. Therefore, the power stability of the laser (although the beam was diffracted at the AOM twice) is not the reason for the fluctuating resonance peak heights since the fluctuation is much higher for them.

Moreover, the pointing stability directly in front of the cavity was measured with a *Thorlabs BC106N-VIS/M* beam profiler with *C-WAVE #023* at a wavelength of  $\lambda_0 = 1045$  nm and is presented in figure (46).



Figure 46: Pointing stability in front of the cavity.

Although the beam pointing exhibits slow drifts of about  $65 \,\mu\text{m}$  over 8 hours<sup>18</sup> the short time stability looks well enough to exclude a pointing stability problem to explain the fluctuating heights of the cavity transmission peaks.

Thus, only the laser beam of C-WAVE #023 (e.g. in a potential multi-mode operation or if unusual high frequency drifts are the case) or the FPI itself can cause the observed fluctuations of the transmission peak heights.

To exclude an influence of the used laser a measurement was done with another laser (ALS seed laser with stated linewidth of 50 kHz). This is shown in figure (47) where the laser beam was incident directly on the FPI and the two scan velocities v = 2 FSR/s and v = 18 FSR/s were chosen.

<sup>&</sup>lt;sup>18</sup>This is not shown here, since in this case only the short time stability is relevant because the cavity field build up time is in the order of few  $\mu$ s.



Figure 47: Transmission fringes taken with the ALS seed laser incident directly on the FPI for the two scan velocities v = 2 FSR/s and v = 18 FSR/s.

Here, for a low scan rate the resonances deviate extremely and for a high scan rate much less from the theoretically expected signal curve. This measurement yields that C-WAVE does not cause the observed resonance heights fluctuations. But, in addition, a high influence of the scan rate is found. To further investigate this dependence, C-WAVE was used with the setup in the path, whose influence was excluded, and resonances were taken for the three scan rates 5 FSR/s, 15 FSR/s and 29 FSR/s. This is presented in figure (48).



Figure 48: Comparison of the transmission fringes taken with C-WAVE with the whole setup in the path for the scan velocities 5 FSR/s, 15 FSR/s and 29 FSR/s.

This measurement yields a definite influence of the scan speed. With higher scan rate, the number of peaks decreases, as in the case of using the *ALS seed laser*. This is probably the case, since with increasing scan rate the time of the overlap between laser lineshape and FPI transmission fringe decreases. Thus,

# 3 LINEWIDTH MEASUREMENT WITH A FABRY-PÉROT INTERFEROMETER USING A LENGTH MODULATION TECHNIQUE

potentially present mechanical instabilities of the FPI would have a decreased influence. Nevertheless, according to figure (41), it is not possible to simply use higher scan rates, since the peak height fluctuations are still far above the desired peak height stability between individual measured resonances to apply the cavity length modulation technique in order to determine the laser's linewidth.

Therefore, a much more stable cavity is to be built.

There was a last measurement to be sure about the interpretation of an unstable cavity leading to the observation of multiple peaks instead of one, as theoretically expected.

Thereby, the cavity input mirror mount was exchanged by a much more stable configuration as shown in figure (49). The cavity input mirror mount used first, was in parts self-built to get a mount that is movable parallel to the optical desk by using the brass screws for a more convenient and precise adjustment of the cavity input mirror mount.

In addition, the output mirror mounted in the cavity output mirror mount, that is affixed to the piezo [see figure (35)] is encapsulated between to washers in this cavity output mirror mount. Here, a third washer was added, to hold this mirror more tightly.



Figure 49: Cavity input mirror holder before (left) and after (right) it was exchanged.

As the measurement depicted in figure (50) shows, the increase of mechanical stability leads to the observation of resonances that are much more consistent with the theory. Thereby, this measurement is to be compared with the resonances shown in figure (44, left).



Figure 50: Single resonance after optimisation of the cavity input mirror mount and the piezo mirror mount.

Probably, the bad mechanical stability of the cavity mirror mounts, especially of the cavity input mirror mount used before the mechanical optimisation, led to instabilities of the mirror focus positions with respect to the FPI's optical axis, that led to the excitation of higher order modes exhibiting other resonance frequencies and took away power from the main resonances.

Moreover, the cavity decay time was remeasured with this now more stable FPI to explain the deviation between the measured cavity mirror reflectivity ( $R = 0.999556 \pm 0.000011$ ) and the one stated by the manufacturer (R = 0.99995). The remeasurement, where only a few measurements were done<sup>19</sup> in order to estimate the decay time after the stability improvements, yields a decay time of at least  $25 \,\mu$ s. This corresponds to mirror reflectivities of at least R = 0.999949 that is in agreement with the stated reflectivity.

Therefore, the instability of the FPI during the prior shown measurements caused this discrepancy.

Nevertheless, a complete redesign of the used cavity with respect to mechanical stability is necessary and it is worth to be done due to the promising results already achieved. Especially, since the previously estimated resolution of 58 kHz [given by equation (86)] is reduced down to 7 kHz if considering the decay time after the mechanical optimisation.

In addition, there was a delayed self-heterodyne interferometry measurement of the linewidth at laser wavelengths in the interval from  $\lambda_0 = 1090 \text{ nm}$  to  $\lambda_0 = 1280 \text{ nm}$  as presented in figure  $(51)^{20}$ .

 $<sup>^{19}</sup>$ Unfortunately, there was not enough time to do accurate measurements again with high averaging and redo the data analysis with a decay time value after the stability improvements.

 $<sup>^{20}</sup>$  The setup was built by K. Fuchs, A. Becker and P. Baum (Department of Technological Physics, Institute of Nanostructure Technologies and Analytics, Kassel University).



Figure 51: Experimental setup of the DSHI experiment. The following symbols are used: AOM (acoustooptic modulator), AOMC (AOM controller), ESA (electrical spectrum analyser), OI (optical isolator), P (polariser) and PD (photo diode).

According to chapter (2.6), the light of the tested laser was coupled into an optical fibre, first. To prevent feedback from the setup, that could influence the lasers output spectrum, an optical isolator (OI, *Thorlabs IO-H-1550FC*) with a reduction of back reflection by 36 dB was used. Afterwards, the beam was split into to paths by means of a 50:50 fibre coupler, whereby the unused port was terminated by 20 dB with an attenuator.

On the first path, a frequency shift of 80 MHz was introduced to the laser light by an acousto-optic modulator (AOM, AA AA.MT80-MIR30-Fio-PM0,5-J1-S-VSF), which was driven by a IntraAction Corp. ME-80 controller (AOMC) with 80 MHz output frequency.

The second path served as delay line with length of 10 km realised by a long optical fibre (*Corning SM Optical Fiber ITU-T G.652.D*). Besides, a polarisation controller (P) was in the path<sup>21</sup>, since only equally polarised light can be used for recording the beat note.

Then, the two paths were combined again with another 50:50 fibre coupler. Here, the unused port was attenuated by 20 dB, too. Therefore, the frequency shifted light of the laser was superimposed with a not frequency shifted but delayed version of it. Hence, a beat note of two quasi independent lasers occurred, that was detected by a fast photo diode (PD, *Thorlabs DET01CFC*) with a bandwidth of 1.2 GHz. In addition, the intensity incident on the PD was measured with a powermonitor (*EigenLight Power Monitor* 410).

Moreover, after filtering out the DC-part of the PD's signal by means of a bias tee from *Picosecond Pulse Labs*, the signal was fed into an electrical spectrum analyser (ESA, *Agilent Technologies N9010A*).

As tested laser Hübner C-WAVE #021 was used.

The measurement data was taken by scanning the beating spectrum with the ESA from 79 MHz to 81 MHz within a sweep time of 0.1 ms (scan rate =  $20 \frac{\text{GHz}}{\text{s}}$ ) and was averaged 100 times. In addition, the individual spectra were analysed by a *Matlab* routine. Such a spectrum is depicted in figure (52) with a convolution of Gaussian and Lorentzian lineshape (Voigt profile) fitted to the data.

 $<sup>^{21}</sup>$ With this kind of polariser it was possible to turn the polarisation plane of the laser light, too.



Figure 52: DSHI measurement: Recorded beating spectrum at 1270 nm normalised to its maximum. ESA sweep time: 0.1 ms, ESA bandwidth: 79 MHz - 81 MHz, ESA scan rate:  $20 \frac{\text{GHz}}{\text{s}}$ , averaging: 100. The red line indicates the Voigt profile as convolution of the underlying Lorentzian profile (blue line) and underlying Gaussian profile (green line), respectively.

Obviously, the Voigt-profile does not describe the lineshape well. Therefore, in order to indicate the FWHM linewidth the full width at the  $-3 \, dB$  level was taken directly. The linewidths measured in this way are shown in figure (53). Thereby, the indicated uncertainty is given by equation (57):

$$R_{\rm DSHI} = \frac{1}{\tau_{\rm delay}} = \frac{1}{48\,\mu\rm{s}} = 21\,\rm{kHz}\;. \tag{87}$$



Figure 53: DSHI measurement: Experimentally obtained linewidths depending on the wavelength. ESA sweep time: 0.1 ms, ESA bandwidth: 79 MHz - 81 MHz, ESA scan rate:  $20 \frac{\text{GHz}}{\text{s}}$ , averaging: 100.

The DSHI measurement yields linewidths between  $\Delta \nu_{0,\text{DSHI}} = (283 \pm 21) \text{ kHz}$  and  $\Delta \nu_{0,\text{DSHI}} = (399 \pm 21) \text{ kHz}$  in the wavelength interval from 1090 nm to 1280 nm depending on the wavelength.

Moreover, it should be outlined again, that the correct delay length has to be chosen. Any other delay length has a strong influence on the measured linewidth [37]. In figure (54) the necessary delay length of the optical fibre depending on the linewidth is shown.



Figure 54: DSHI measurement: Necessary fibre length. Calculated for a refractive index of  $n_{\rm core} = 1.41$ using the relations  $\Delta \nu_0 = t_{\rm coh}^{-1}$  between the laser linewidth  $\Delta \nu_0$  and the coherence time  $t_{\rm coh}$  and  $t_{\rm coh} = l_{\rm coh}c_{\rm fibre}^{-1}$  between coherence length  $l_{\rm coh}$  and time  $t_{\rm coh}$  with the speed of light in the fibre  $c_{\rm fibre}$ , respectively. The length of the fibre used in the experiment was 10 km.

According to necessary fibre lengths empirically obtained, in practice the necessary fibre length is roughly four times the theoretically calculated value [stated in figure (54)]. Considering additionally the measured linewidths [see figure (53)], a fibre length of about 4 km was necessary, but a fibre with length of 10 km was used.

In [37] it is discussed, that a longer delay length than necessary leads to an increase of the measured linewidth due to noise introduced by the long optical fibre. For example, a initially measured linewidth of 1.000 MHz for a delay length of 1 km resulted in 1.423 MHz linewidth in case of 5 km delay.

Therefore, it is probably, that the linewidths of C-WAVE measured with DSHI are increased due to noise introduced by the long fibre. Moreover, this also explains why the lineshape shown in figure (52) seems to be broadened (strong Gaussian part).

In addition, the OPO resonator's optical round-trip path length (of C-WAVE) is controlled by a piezo electric transducer, since one of the resonator mirrors is mounted on this piezo. The piezo varies the resonator length in order to achieve a constant carrier frequency of emitted laser radiation by using the Pound-Drever-Hall stabilisation technique. Moreover, after the DSHI measurement it was observed that there were jumps in the voltage applied at the piezo leading to spontaneous shifts of the central emission wavelength in the order of a few hundreds of kHz.

If such a shift occurs between the measurement of two individual beating spectra, they should not be affected. But if the laser frequency changes by many kHz during the time the beating spectrum is scanned by the ESA and the shift is not discrete and instant with respect to the corresponding timescale (the time of recording one beating spectrum), an increased linewidth should occur. Besides, the lineshape should be flattened at the top.

Since the voltage jumps occurred on timescales shorter than the measurement time for one spectrum, this consideration yields a strongly broadened and at the top flattened lineshape as expected beating spectrum and this describes the recorded beating spectrum [shown in figure (52)] well.

Thus, the linewidths obtained by DSHI are probably broader than in reality.

Now, although an accurate calculation of the linewidth of emitted laser radiation of C-WAVE is not possible due to the complexity of the OPO-system, an estimation is done. Thereby, a Singly-resonant continious-wave optical parametric oscillator [see chapter (2.2)] is considered, where the signal is resonant and the idler is non-resonant and coupled out. Moreover, the OPO-resonator is composed of a non-linear crystal used for the parametric interaction, two intra-cavity etalons ET1 and ET2, respectively, and four mirrors that realise a ring cavity. Thereby, the OPO is similar to the model depicted in figure (5) but with two etalons integrated.

The linewidth of the outcoupled idler light is determined by the following quantities:

- The gain curve width  $\Delta \nu_{\text{Gain}}$  of the non-linear parametric interaction,
- the bandwidth  $\Delta \nu_{c,ET1}$  of the intra-cavity etalon ET1 (regarding the resonant signal beam),
- the bandwidth  $\Delta \nu_{c,ET2}$  of the intra-cavity etalon ET2 (regarding the resonant signal beam),
- the bandwidth  $\Delta \nu_{\rm c.res}$  of the ring-resonator cavity (regarding the resonant signal beam) and
- the linewidth  $\Delta \nu_{0,\text{pump}}$  of the pump laser,

those are consecutively discussed, now.

Firstly, the width  $\Delta\nu_{\text{Gain}}$  of the gain curve, resulting from the quasi-phase-matching in the non-linear crystal for a given temperature, has to be broad in relation to the bandwidths of the intra-cavity etalons  $\Delta\nu_{c,\text{ET1}}$  and  $\Delta\nu_{c,\text{ET2}}$ , respectively, the bandwidth of the resonator cavity  $\Delta\nu_{c,\text{res}}$  and the linewidth of the pump laser  $\Delta\nu_{0,\text{pump}}$ , since the central wavelength of the outcoupled idler light is tunable within the gain curve without changing the crystal temperature and, therefore, the gain curve itself, over more than 100 GHz.

Secondly, the bandwidth of the ring-resonator cavity  $\Delta\nu_{\rm c,res}$  regarding the resonant signal beam is determined by the optical round-trip path length  $L_{\rm opt,res}$  of the cavity and the mirror reflectivities of the individual mirrors  $R_{\rm res,i}$ . Since the mean reflectivity of the mirrors  $R_{\rm res,avg}$  is given by the product of the individual reflectivities,  $R_{\rm res,avg} = 0.999978 \cdot 0.999978 \cdot 0.999950 \cdot 0.999976 = 0.999882$  results [17]. Here as following, the calculation is exemplarily done for an idler wavelength of 1185.0 nm corresponding to a signal wavelength of 965.4 nm. According to equation (31), this yields for the finesse  $F_{\rm res} = 26622$ . Moreover, the round-trip length  $L_{\rm opt,res} = 38.9$  cm corresponds to a free spectral range [eq. (33)] of  $\Delta\nu_{\rm FSR,res} = 771$  MHz that, in return, leads to a bandwidth of  $\Delta\nu_{\rm c,res} = 28.9$  kHz.

Thirdly, the effective bandwidth  $\Delta \nu_{c,i}$  of the intra-cavity etalon i (i = ET1, ET2) depends on the average number of round-trips of the light in the cavity  $N_{\rm rt}$ , since with each round-trip the light is affected by the etalon again [14] [5]. Thereby, if p is the number of passes, the relation between the single-pass bandwidth  $\Delta \nu_{c,i}(p = 1)$  and the bandwidth after p passes  $\Delta \nu_{c,i}(p)$  is given by [14]:

$$\Delta \nu_{\rm c,i}(p) = \frac{1}{\sqrt{p}} \Delta \nu_{\rm c,i}(p=1) .$$
(88)

Hence, first the number of round-trips  $N_{\rm rt}$  has to be estimated. Neglecting any losses and considering only the empty cavity,  $N_{\rm rt}$  is given by [5]:

$$N_{\rm rt} = \frac{F_{\rm res}}{\pi} = \frac{26622}{\pi} = 8474 \;. \tag{89}$$

Obviously, the value for  $N_{\rm rt}$  should be to high since, e.g. there is additional absorption at the crystal, so  $N_{\rm rt}$  can be seen as an upper limit to the number of round-trips.

In addition, the single-pass bandwidth  $\Delta\nu_{c,ET1}(p=1)$  is to be calculated. Thereby, the mirror reflectivities of the air-spaced etalon ET1 are equal and given by  $R_{\rm ET1} = 0.5$ , leading to  $F_{\rm ET1} = 4.44$ . Moreover, the spacing between the etalon plates is changeable by means of a piezo electric transducer and, thus, the free spectral range varies between  $\Delta\nu_{\rm FSR,ET1} = 200 \,\text{GHz}$  and  $\Delta\nu_{\rm FSR,ET1} = 250 \,\text{GHz}$ , where in the following the average value  $\Delta\nu_{\rm FSR,ET1} = 225 \,\text{GHz}$  is used for calculation. By this, a single-pass bandwidth of  $\Delta \nu_{\rm c,ET1}(p = 1) = 50.7 \,\text{GHz}$  results. Besides, the bandwidth considering the mean number of cavity transits  $N_{\rm rt}$  is calculated to be:

$$\Delta\nu_{\rm c,ET1} := \Delta\nu_{\rm c,ET1}(p = N_{\rm rt}) = \frac{1}{\sqrt{N_{\rm rt}}} \cdot \Delta\nu_{\rm c,ET1}(p = 1) = \frac{1}{\sqrt{8474}} \cdot 50.7 \,\rm{GHz} = 551 \,\rm{MHz} \,. \tag{90}$$

As already mentioned,  $N_{\rm rt}$  can be seen as an upper limit for the cavity round-trips. Therefore,  $\Delta \nu_{\rm c,ET1}$  is probably broader in reality.

Fourthly, the intra-cavity etalon ET2 has to be treated as ET1.

Since, ET2 is a solid 25  $\mu$ m thick uncoated YAG etalon, the reflectivity of the endfacets is given by the refractive index step between air ( $n_{\rm air} = 1.000277$ , from [38]) and YAG ( $n_{\rm YAG} = 1.833$ , from [39]). According to the freshel formula [10], the reflectivity is:

$$R_{\rm ET2} = \left| \frac{n_{\rm air} - n_{\rm YAG}}{n_{\rm air} + n_{\rm YAG}} \right|^2 = \left| \frac{1.000277 - 1.833}{1.000277 + 1.833} \right|^2 = 0.086 .$$
(91)

For such a low reflectivity, the formula for the finesse (31) is no longer valid, since it is an approximation for reflectivities close to one<sup>22</sup>. In addition, the exact formula for the Airy finesse, that is given by [26]:

$$F = \frac{\Delta\nu_{\rm FSR}}{\Delta\nu_{\rm c}} = \frac{\pi}{2} \left[ \arccos\left(\frac{1 - \sqrt{R_1 R_2}}{2 \cdot (R_1 R_2)^{\frac{1}{4}}}\right) \right]^{-1}$$
(92)

has no solution for reflectivities smaller than given through  $\sqrt{R_1R_2} = 0.172$ . The fundamental reason is, that the concepts of linewidth and finesse of the Airy distribution break down at this point at that the finesse equals one [26], because the linewidth would be broader than the free spectral range if the reflectivity would be lowered further.

Hence, the second intra-cavity etalon ET2 is compared with the first one ET1 to discuss its influence on the linewidth.

With thickness and refractive index of the YAG etalon a free spectral range of  $\Delta \nu_{\text{FSR,ET2}} = 3271 \text{ GHz}$  follows. Therefore, considering the first equality in equation (92), the linewidth of the intra-cavity etalon ET2 is much broader than the linewidth of ET1, since, even if the reflectivities of both etalons would be equal (resulting in the same finesse), the linewidth of ET2 would be  $\frac{\Delta \nu_{\text{FSR,ET2}}}{\Delta \nu_{\text{FSR,ET1}}} = \frac{3271 \text{ GHz}}{225 \text{ GHz}} = 14.5$  times broader than the one of ET1. Hence, the second etalon is negligible in this discussion.

Indeed, even the bandwidth of the first etalon ET1 is broad in comparison with the cavity bandwidth. Thus, both etalons ET1 and ET2 do not affect the linewidth of the outcoupled idler beam and only serve the purpose, to tune the center frequency of the outcoupled light.

Fifthly, the influence of the pump laser with respect to its linewidth  $\Delta \nu_{0,\text{pump}}$  has to be discussed.

It affects the linewidth of the outcoupled idler light, since the idler linewidth is given by the convolution of the resonating signal and singly passing pump beam [5]. The considerations done before yield for the signal a linewidth  $\Delta \nu_{0,\text{signal}}$  of roughly the cavity bandwidth  $\Delta \nu_{c,\text{res}}$ , since the etalons have very broad bandwidths in comparison with the cavity bandwidth. Thus,  $\Delta \nu_{0,\text{signal}} \approx \Delta \nu_{c,\text{res}} \approx 28.9 \text{ kHz}$ . In addition, the linewidth of the pump laser *Cobolt Samba* (cw diode-pumped solid-state laser, 532 nm, 1500 mW) is specified to be  $\Delta \nu_{0,\text{pump}} < 1 \text{ MHz}$ , but according to the manufacturer it should be considerably smaller. Moreover, the lineshape should be Lorentzian shaped due to homogeneous thermal phonon broadening [9].

Referring to chapter (2.1), the linewidth of the convolution of two Lorentzians is given by the sum of the individual linewidths. Moreover, the lineshape of the cavity is given by an Airy function, but it can be approximated with a Lorentzian function, because the cavity mirror reflectivities are very high [40]. Thus, at 1185 nm the linewidth of the idler and, therefore of the output beam of C-WAVE,  $\Delta \nu_0$ , is:

$$\Delta \nu_0 = \Delta \nu_{0,\text{pump}} + \Delta \nu_{0,\text{signal}} = \Delta \nu_{0,\text{pump}} + 28.9 \,\text{kHz} \,. \tag{93}$$

Considering the linewidth measured with DSHI at 1185 nm of  $(308 \pm 21)$  kHz [see figure (53)], it is not meaningful to calculate the idler linewidth with the specified value of  $\Delta \nu_{0,\text{pump}} \approx 1$  MHz. Instead, for each wavelength measured [shown in (53)] the signal linewidth  $\Delta \nu_{0,\text{signal}}$  is calculated as described above

 $<sup>^{22}</sup>$ Nevertheless, formula (31) was used to calculate the finesse for the mirror reflectivity  $R_{\rm ET1} = 0.5$ , because the error made here is already small in this case.

and plotted in figure (55). Then, using equation (93), the pump laser linewidth is calculated for each wavelength by subtracting the calculated signal linewidth from the measured idler linewidth. This leads to a pump linewidth of  $\Delta \nu_{0,\text{pump}} = (258.7 \pm 36.0)$  kHz where as uncertainty the standard deviation is taken. In a self-consistent way the individual calculated signal linewidths are added to the the averaged pump linewidth of  $\Delta \nu_{0,\text{pump}} = (258.7 \pm 36.0)$  kHz and plotted in figure (55). This approach is self-consistent, since for nearly all wavelengths the measured and the calculated idler linewidths match within their uncertainties<sup>23</sup>. Therefore, the theoretically calculated and experimentally measured values are self-consistent. That proves that it is a valid approximation to consider only the four resonator mirrors and to neglect both etalons, any absorption, e.g. in the non-linear crystal and acoustical and mechanical noise, that affects the cavity length and therefore the bandwidth and linewidth of the resonating signal light. At least the last influence should be strongly reduced because of the used Pound-Drever-Hall technique that stabilises the cavity length [5].



## Comparison of measured and calculated idler linewidth

Figure 55: DSHI measurement: Comparison of measured and calculated idler linewidths. In addition, the calculated linewidths of the resonant signal beam are depicted, too.

Unfortunately, it is not possible to compare the linewidths measured with DSHI with the linewidths obtained from the own setup, since no successful measurement with the cavity length modulation technique could be done during this thesis.

#### 3.3 Summary

#### 3.3.1 Cavity characterisation measurement

The cavity ring-down measurement was done in order to determine the FPI's DT to calculate the FPI's finesse and mirror reflectivities by using the measured FSR.

 $<sup>^{23}</sup>$ The used *C-WAVE* was not equipped with a wavemeter. In this case a much higher uncertainty of the emitted center wavelength of 2 nm is given, but the reflectivities are taken from the data sheets of the mirrors for the configured wavelengths. Here, at least, it is possible for the wavelength 1280 nm to choose mirror reflectivities of the adjacent wavelength 1278 nm that result in a consistent calculated idler linewidth.

It leads a DT  $\bar{\tau}_c = (2.434 \pm 0.055) \,\mu$ s in case of using the AOM to scan the frequency with 10 times averaging and  $\tau_{c, \text{ piezo}} = (2.85562 \pm 0.00062) \,\mu$ s in the case of using the piezo to scan the frequency with 500 times averaging, respectively. Both results do not agree within their uncertainties. This is probably caused by the low averaging in case of the measurement with the AOM in combination with the observed mechanical instability of the built FPI<sup>24</sup>. This was validated by a measurement after improving the FPI's mechanical stability. Here, much higher decay times of at least 25  $\mu$ s were observed leading to calculated mirror reflectivities that agree with the by the manufacturer stated ones by using equations (31), (32) and the measured FSR. Therefore, after optimisation this measurement yields valid results.

The FSR measurement yields  $\overline{\Delta\nu}_{\text{FSR}} = (393.9 \pm 9.6) \text{ MHz}$  and is in compliance with the result from measuring the mirror spacing  $d_{\text{m}} = (38.0 \pm 0.1) \text{ cm}$  directly and calculating with  $d_{\text{m}}$  the FSR leading to  $\Delta\nu_{\text{FSR, ruler}} = (394.3 \pm 1.0) \text{ MHz}.$ 

In addition, the results of both measurements exhibit only a weak dependence on temperature changes. Therefore, it is not necessary to characterise the cavity for a set of temperatures or even for each measurement, as originally planned.

Moreover, using the results of cavity ring-down and cavity FSR measurement leads to a finesse of  $F = 7068 \pm 174$ . This is far below the expected value of F = 62830 if considering the stated reflectivities of the resonator mirrors [with equation (31)] and results from the low measured DT reduced by the mechanical instabilities of the FPI itself<sup>25</sup>. Considering the DT after mechanical optimisation a finesse of at least F = 61873 results.

#### 3.3.2 Cavity length modulation measurement

Within this thesis, it did not function to measure linewidths with the cavity length modulation technique due to high fluctuations of the cavity transmission fringe heights that is the value measured for a varied scan speed to obtain the laser's linewidth.

Here, the investigation of this problem by observing the transmitted intensity for different measurement conditions yield the low mechanical stability of the FPI itself as the fundamental reason. Thereby, all components of the experimental setup, except the FPI, could be excluded after another by using a second laser, varying the scan speed of the piezo, in- and excluding the own setup with respect of the AOM double-pass configuration in the path and checking the power and pointing stability in front of the FPI. Finally, this result was verified by optimising the stability of both cavity mirror mounts and, then, observing the transmitted intensity resulting in resonances much more in compliance with theoretically expected ones.

Indeed, the FPI's estimated resolution before the optimisation of mechanical stability was already in the order of 60 kHz. After optimisation, even a value of 7 kHz is estimated. Here, no uncertainties of the measured magnitude of the fringe are included that mainly originate from the mechanical instability of the built FPI. Nevertheless, the already achieved resolution is very promising. Therefore, it is worthwhile to build up a much more stable FPI after this thesis. Then, it should be possible to measure linewidths in the order of 100 kHz especially because there is principally no actively length stabilised FPI necessary to apply the cavity length modulation technique.

In addition, the linewidth of the tested laser<sup>26</sup> was measured with an already existing delayed selfheterodyne interferometry setup. This measurement yields linewidths from  $\Delta \nu_{0,\text{DSHI}} = (283 \pm 21) \text{ kHz}$  to  $\Delta \nu_{0,\text{DSHI}} = (399 \pm 21) \text{ kHz}$  depending on the emission wavelength of the used laser.

Moreover, the elements determining the linewidth of the optical parametric oscillator process used in C-WAVE were theoretically investigated and lead to results that are very in compliance with the measured linewidths obtained by the delayed self-heterodyne interferometry measurement. Unfortunately, the exact

 $<sup>^{24}</sup>$ Obviously, the uncertainties of these values are chosen too small, since they do not agree within their uncertainties. Here, statistical uncertainties are included and influences of length drifts due to temperature changes are dropped because it was shown that they are small. Not included is the mechanical instability of the FPI that is unquantifiable.

<sup>&</sup>lt;sup>25</sup>The mirror reflectivity of R = 0.99995 stated by the manufacturer is a value that has to be fulfilled. Therefore, the mirrors are built to have a theoretical reflectivity of R = 0.999971 corresponding to a theoretical finesse of F = 108330.

 $<sup>^{26}</sup>$ Hübner C-WAVE #021

linewidth of the OPO's pumplaser is not known. Therefore, these results support the measured linewidths but do not serve as a reference.

# 4 Conclusion and outlook

The cavity characterisation experiments are working and yielding reasonable results. Therefore, it is possible to use these measurements to characterise cavities.

The linewidth measurement using the cavity length modulation technique could not be applied due to the mechanical instability of the built Fabry-Pérot interferometer. After this thesis, a much more stable cavity will be used to measure linewidths with this technique due to the promising results already achieved. Here, it is planned to use a cavity as built in in C-WAVE, since it is verified that here no stability problems are the case. This cavity will be wavelength dependently characterised by the cavity characterisation measurements. Then, as soon as the linewidths with this cavity and the cavity length modulation technique are successfully measured in the infrared region, the extension to the visible region will be done. Essentially, for this realisation only the cavity mirrors and some other components like the AOM are to be exchanged.

Moreover, as soon as linewidths measured with the cavity length modulation technique are available in the region from 1090 nm to 1280 nm, it is still possible to compare them with the results obtained by the delayed self-heterodyne interferometry experiment, as originally intended.

## 5 Appendix

#### 5.1 Numerical calculation of the magnitude of the fringe

The correct numerically solving of equation (41) is of fundamental importance. Therefore, it is discussed in more detail. The Magnitude of the fringe is given by:

$$MF(l) = \frac{I_{\text{out}}(l)}{I_0} = T^2 \int_0^\infty \mathrm{d}\nu \frac{1}{\pi} \frac{\frac{\Delta\nu_0}{2}}{(\nu - \nu_0)^2 + (\frac{\Delta\nu_0}{2})^2} \cdot R^{2[l+\delta(\nu)]} \cdot \left| \sum_{n=-[l+\delta(\nu)]}^\infty R^n \exp\left\{i2\pi\nu t_r \frac{v}{c_0} n^2\right\} \right|^2 .$$
(94)

The parameter T (mirror transmittance), R (mirror reflectivity),  $t_r$  (cavity round-trip time),  $\nu_0$  (laser frequency) are given by the experiment.

l is the time t normalised to the round-trip time  $t_{\rm r}$ :

$$l = \frac{t}{t_{\rm r}} \,. \tag{95}$$

Since the round-trip time  $t_r = (2.539 \pm 0.062)$  ns is small in comparison to the decay time  $\tau_c = (2.85562 \pm 0.00065) \,\mu$ s, l is taken as an integer.

Moreover,  $\delta(\nu)$  is the dimensionless time difference between resonant instances for the frequencies  $\nu$  and  $\nu_0$  [6]:

$$\delta(\nu) = -\frac{c_0(\nu - \nu_0)}{2\nu\nu_0} = -\frac{c_0}{2\nu\nu_0} \cdot \nu + \frac{c_0}{2\nu} , \qquad (96)$$

with the speed of light  $c_0$  and the mirror velocity<sup>27</sup> v given by the experiment. Thereby,  $\delta(v)$  is taken as an integer, too.

The sum is solved for *n*-values from  $-[l + \delta(\nu)]$  to  $n_{\max}$ , where  $n_{\max}$  is obtained from the condition:

$$R^{n_{\max}} \stackrel{!}{=} \alpha_{\min} \approx 10^{-6} . \tag{97}$$

 $\alpha_{\min}$  was chosen by the condition, that the with equation (94) obtained *MF*-values vary on digits that are not of importance any more.

Moreover, the integral is solved for frequencies in the interval from the laser linewidth  $\nu_0$  minus several linewidth to the laser linewidth plus the same amount of linewidths. Again, here it was tested how large this interval has to be in order to obtain results only varying on digits that do not matter.

<sup>&</sup>lt;sup>27</sup>The mirror velocity is in the order of a few  $\frac{\mu m}{s}$ .

# Abbreviations

Abbreviation	Meaning
А	Aperture
AOM	Acousto-optic modulator
AOM-DPC	AOM double-pass configuration
AOMC	AOM controller
ASG	AOM stop-signal generator
BCL	Bi-convex lens
BD	Beam dump
BS	Beam splitter
CLMT	Cavity length modulation technique
CRDM	Cavity ring-down measurement
DSHI	Delayed self-heterodyne interferometry
DT	Decay time
ESA	Electrical spectrum analyser
FSM	Finite state machine
FSR	Free spectral range
FWHM	Full-width at half-maximum
HWP	Half-wave plate
IBUF	Input buffer
М	Mirror
MM	Mode-matching
OBUF	Output buffer
OI	Optical isolator
OPO	Optical parametric oscillator
Р	Polariser
PBS	Polarising beam splitter
PC	Personal computer
PCL	Plano-convex lens
PD	Photo diode
PET	Piezo electric transducer
PSD	Power spectral density
QPM	Quasi-phase-matching
QWP	Quarter-wave plate
Scope	Oscilloscope
SR-OPO	Singly-resonant OPO
TM	Tiltable mirror
VCO	Voltage controlled oscillator

Table 1: Table of abbreviations.

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## Eigenständigkeitserklärung

Hiermit versichere ich, dass ich die vorliegende Masterarbeit selbständig und ohne unerlaubte Hilfe angefertigt und andere als in der Masterarbeit angegebene Hilfsmittel nicht benutzt habe. Alle Stellen, die wörtlich oder sinngemäß aus veröffentlichten oder unveröffentlichten Schriften entnommen sind, habe ich als solche kenntlich gemacht. Kein Teil dieser Arbeit ist in einer anderen Abschlussarbeit verwendet worden.

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